MODULE

ON

FRACTIONS

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J.R.B.
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Introduction

Many quantities and objects in the world are expressed in terms of fraction. For instance, most recipes list quantities on fractions. Fractions are also significant in problem solving.

This module deals with fractions. You will learn how to order and compare fractions. Finally, you will also find out how to perform operations on fractions and apply these basic concepts to problem solving.

The module is divided into three lessons, namely: **Lesson 1** –  *Rational Numbers with subtopics, Changing Fractions to Decimals and Changing Decimals to Fractions*; **Lesson 2** –  *Rational Numbers on the Number Line*, and **Lesson 3** –  *Operations with Fractions with subtopics Addition and Subtraction and Multiplication and Division*. 
General Instructions

After going through this module, you should be able to:

- Define rational numbers
- Identify some forms of rational numbers
- Order fractions
- Compare fractions
- Simplify fractions
- Perform operations on fractions; and
- Solve problems involving fractions

This is your guide for your proper use of the module:

1. Read the items in the module carefully.
2. Follow the directions as you read the materials.
3. Answer all the questions that you encounter. As you go through the module, you will find help to answer these questions.
4. You must be patient and industrious in doing suggested tasks to be successful in undertaking this module.
5. Take your time to study and learn.
Lesson 1 Rational Numbers

At the end of the lesson, the students are expected to:

a. Define rational numbers.

b. Change fractions to decimals and vice – versa.

Concepts

Integers alone are not sufficient to represent all the numbers we encounter in real life. For instance, if a P400.00 contest prize is divided among 3 members, each will get \( \frac{400}{3} \) or 133.333… These numbers are no longer integers. Fractions and repeating or repeating decimals are part of a larger set of numbers called rational numbers.

The phrase \( b \neq 0 \) is an important restriction because division by 0 is not allowed. There is no specific quotient that can answer \( \frac{a}{b} \) if \( b = 0 \).

For example,

The problems \( 5 \times 0 = 0 \) cannot be taken to mean as \( 0 \div 0 = 2 \);

\( 25 \times 0 = 0 \) cannot be taken to mean as \( 0 \div 0 = 25 \).

Not all decimals can be changed to fractions. For example, 0.50 can be written as \( \frac{50}{100} \) or \( \frac{1}{2} \), 0.55… can be written as \( \frac{5}{9} \), but the numbers \( \sqrt{2}, \sqrt{11}, \pi \) cannot be written in the form \( \frac{a}{b} \). These numbers are called irrational numbers.
Did you know?

A **rational number** is a number that can be expressed as a quotient of two integers. The integer **a** is the numerator while the integer **b**, is the denominator, which cannot be **0**.

**Example:**

\[
\frac{3}{4} \text{ numerator}
\]

\[
\frac{4}{4} \text{ denominator}
\]

**Other examples are:**

\[
\frac{1}{2}, \frac{6}{3}, -\frac{20}{4}, -\frac{5}{7}, \frac{6}{-6}, 4, 5, \sqrt{9}, \sqrt[3]{8}
\]

An **irrational number** is a number that cannot be expressed as a quotient of two integers. Every irrational number may be represented by a decimal that neither repeats nor ends.

**Examples:**

\[
\pi \text{ (pi) and } \sqrt{2}
\]

\[
\pi = 3.14159265 \ldots
\]

\[
\sqrt{2} = 1.4142135 \ldots
\]
1.1 Changing Fractions to Decimals

To convert written fractions to decimal fractions, simply perform the operation that is indicated by the fraction bar.

**Exploration**

Consider these examples:

**Example 1**

COMMON ERROR !!!

\[
\begin{array}{c}
\text{Interchanging} \\
3 \text{ and } 5 \text{ is} \\
\text{incorrect.}
\end{array}
\]

\[
\begin{array}{c}
3 \overset{\_}{\big|} 5.00 \\
3 \overset{\_}{\big|} 3.00 \\
2 \overset{\_}{\big|} 18 \\
18 \overset{\_}{\big|} 20 \\
18 \overset{\_}{\big|} 2
\end{array}
\]

When changing fraction to decimal, it is a mistake to put the denominator as the dividend and the numerator of the fraction as the divisor. Usually, students fail to recognize that **dividing fractions is the same with dividing whole numbers**.

**CORRECT ✓**

\[
\begin{array}{c}
3 \overset{\_}{\big|} 3.00 \\
5 \overset{\_}{\big|} 30 \\
0 \overset{\_}{\big|} 0
\end{array}
\]

Numerator should be the dividend and the denominator should be the divisor.
In changing fraction to decimal, it is appropriate to put the numerator as the dividend and the denominator as the divisor, and then follows the process of division.

Therefore, \( \frac{3}{5} = 0.6 \)

Example 2

**COMMON ERROR !!!**

\[
\begin{array}{c}
7 \\
20 \\
\hline
\end{array}
\]

Interchanging 7 and 20 is an error.

\[
\begin{array}{c}
2.857... \\
7 \)
20.000 \\
14 \\
60 \\
56 \\
40 \\
35 \\
50 \\
49 \\
1 \\
\hline
\end{array}
\]

In the example above, the error is 20 becomes the dividend and 7 becomes the divisor.

**CORRECT ✓**

\[
\begin{array}{c}
7 \\
20 \\
\hline
\end{array}
\]

\[
\begin{array}{c}
0.35 \\
20) 7.00 \\
60 \\
100 \\
100 \\
0 \\
\hline
\end{array}
\]

Numerator should be the dividend and denominator should be the divisor.
When changing fraction to decimal, the numerator will serve as the dividend and the denominator is the divisor.

Therefore, \( \frac{7}{20} = 0.35 \).

Example 3.

COMMON ERROR !!!

\[
\begin{array}{c}
1 \\
8
\end{array}
\]

Interchanging 1 and 8 is an error.

\[
\begin{array}{c}
0.125 \\
0.1000
\end{array}
\]

Numerators should be the dividend and the denominator should be the divisor.

The error found above is interchanging the 1 and 8 since 1 must be the numerator and 8 must be the denominator.

CORRECT √

\[
\begin{array}{c}
1 \\
8
\end{array}
\]
The fractions above when changed to decimal numbers give exact values. These are referred to as **terminating decimals**. Other fractions of this type are \( \frac{4}{5}, \frac{3}{4}, \frac{3}{8} \), and so on.

There are times that the repeating digit/digits in the quotient do not appear at once.

**Example 4**

<table>
<thead>
<tr>
<th>COMMON ERROR !!</th>
</tr>
</thead>
</table>
| \[
\begin{array}{c}
\phantom{0}1.75 \\
\hline
4 \overline{)7.00}
\end{array}
\]

Interchanging 4 and 7 is an error.

\[
\begin{array}{c}
\phantom{0}4 \\
3 0 \\
\hline
2 8 \\
2 0 \\
\hline
0
\end{array}
\]

In the example above, the error committed is interchanging 4 and 7 wherein 4 must be the dividend and 7 must be the divisor.
CORRECT ✓

\[
\begin{array}{c}
\frac{4}{7} \\
7)4.000000 \\
\hline \\
35 \\
50 \\
\hline \\
49 \\
10 \\
\hline \\
7 \\
30 \\
\hline \\
28 \\
20 \\
\hline \\
14 \\
60 \\
\hline \\
56 \\
4 \\
\end{array}
\]

\[0.571428\ldots\]

Notice that the remainder 4 is the same as the original dividend, so the decimal will repeat at that point. Thus, the repeating digits are 571428.

\[\frac{4}{7} = 0.\overline{571428}\]

In the examples above, the digit that repeats itself is called the **repetend**, indicated oftentimes with three dots called **ellipsis**, or simply a bar (**vinculum**) over the repetend.
Self – check 1.1

A. Change the given fractions to decimals.

1. \( \frac{2}{5} \)  
2. \( \frac{9}{10} \)  
3. \( \frac{6}{24} \)  
4. \( \frac{5}{25} \)  
5. \( \frac{8}{20} \)

6. \( \frac{3}{8} \)  
7. \( \frac{4}{5} \)  
8. \( \frac{2}{3} \)  
9. \( \frac{5}{6} \)

10. \( \frac{1}{3} \)

B. Show whether the following fractions are terminating or repeating decimals.

1. \( \frac{3}{7} \)  
2. \( \frac{3}{8} \)  
3. \( \frac{9}{20} \)  
4. \( \frac{16}{52} \)  
5. \( \frac{7}{8} \)

6. \( \frac{5}{6} \)  
7. \( \frac{3}{10} \)  
8. \( \frac{4}{11} \)  
9. \( \frac{7}{13} \)

10. \( \frac{5}{7} \)
1.2 Changing Decimals to Fractions

Decimal number is a number which always uses 10 as its base and it is most widely used to solve the complex problems. It is denoted with a dot (.) in between it. This dot is termed as **decimal point**. It is also called **separatrix**. Decimal numbers do not necessarily contain a decimal point; 245, 2.45, and -245 are all decimal numbers.

The unit places of any decimal number are denoted by using positions tenths place, hundredths place, thousands place etc. Here are some illustrations below:

**Representing a Decimal on a Number Line**

To represent a decimal on a number line, divide each segment of the number line into ten equal parts.

**Example.** To represent 8.4 on a number line, divide the segment between 8 and 9 into ten equal parts.

The arrow is four parts to the right of 8 where it points at 8.4.

Likewise, to represent 8.45 on a number line, divide the segment between 8.4 and 8.5 into ten equal parts.
The arrow is five parts to the right of 8.4 where it points at 8.45.

Similarly, we can represent 8.456 on a number line by dividing the segment between 8.45 and 8.46 into ten equal parts.

The arrow is six parts to the right of 8.45 where it points at 8.456.

To convert decimal fractions to written fractions, we take note the place value system. Then, change to lowest terms when possible.

**Exploration**

Study the samples given below.

**Example 1**

**COMMON ERROR !!!**

\[ 0.6 = \frac{1}{6} \]

Thinking that the 6 is in the ones place and writing 1 as the numerator.
In the example above, it’s an error since the student thinks that the 6 is in the ones place. The students failed to familiarize and understand the concept of place value.

**CORRECT √**

0.6 is read as “6 tenths.”

\[
0.6 = \frac{6}{10} = \frac{3}{5} \quad \text{Change to lowest terms}
\]

In the illustration above, it’s the proper way of changing decimal to fraction since the place value of 6 is in the tenths place so the denominator of the fraction must be 10 and then reduces fraction if possible.

**Example 2**

**COMMON ERRORS !!!**

a. \(0.25 = \frac{25}{10}\), thinking that the place value started in “ones place”

b. \(0.25 = \frac{25}{100}\), not reduced to lowest term

**CORRECT √**

0.25 is read as “zero and twenty – five hundredths.”

\[
0.25 = \frac{25}{100} = \frac{1}{4}
\]
Example 3.

**COMMON ERRORS !!!**

a. \(0.125 = \frac{125}{100} = \frac{5}{4}\)
   thinking that the place value
   started in “ones place” is an error

b. \(0.125 = \frac{125}{1000}\), not reduced to lowest term

The illustrated examples above have errors since in
number (1); the student thinks that the place value of
decimals started at ones place and in number (2), the
fraction is not reduced to lowest terms.

**CORRECT √**

\[0.125\] is read as “zero and one hundred twenty-five
thousandths.”

\[0.125 = \frac{125}{1000} = \frac{1}{8}\]

To change decimal to fraction, it is appropriate to
know the place values of decimal numbers. After changing it
a fraction, reduce it to the lowest terms.

From the illustration above, we can say that terminating
decimals can be easily expressed as fractions.

Suppose you are asked to convert a repeating, nonterminating
decimal to fraction form. How will you do it? Here are some rules:

- **If the repetend consists of 1 digit, multiply both sides of
  the equation by 10 and subtract \(n\) from \(10n\).**
Example  Convert 0.888...

0.88: Let $n = 0.888...$ Multiply both sides by 10, since the repetend which is 8 started at the tenth place.

$$10n = 8.88...$$

Subtract $n$ from $10n$, then it becomes

$$10n = 8.88...$$
$$- n = 0.88...$$
$$9n = 8$$

$$n = \frac{8}{9}; \text{ thus, } 0.44... = \frac{4}{9}$$

Example 4: Convert 0.444... to fraction.

**COMMON ERROR !!!**

$$0.444... = \frac{44}{100} = \frac{11}{25}$$

Converting repeating non terminating decimal to fraction is the same with converting terminating decimal to fraction.

The error found in illustration above is the process of changing a non terminating decimal to fraction since it is not the same with changing terminating decimal fraction.

**CORRECT ✓**

$$0.44: \text{ Let } n = 0.44...$$ Multiply both sides by 10, since the repetend which is 4 started at the tenth place.

$$10n = 4.44...$$

Subtract $n$ from $10n$, then it becomes
10n = 4.44...
- n = 0.44...
9n = 4

n = \frac{4}{9}; \text{ thus, } 0.44... = \frac{4}{9}

- If the repetend consists of 2 digits, multiply both sides of the equation by 100 and subtract n from 100n

Example  Convert 0.171717...

Let n = 0.171717...  Let n be the given number
100n = 17.1717...  Multiply both sides of the equation by 100

Subtracting n from 100n we have

100n = 17.1717...
- n = 0.171717...
99n = 17

n = \frac{17}{99}

Therefore, 0.171717... = \frac{17}{99}

Example 5:  Convert 0.151515... to fraction

COMMON ERROR !!!

0.151515... = \frac{15}{100} = \frac{3}{20}  \text{ Converting repeating non-terminating decimal to fraction is the same with converting terminating decimal to fraction.}
The error found in illustration above is the process of changing a non terminating decimal to fraction since it is not the same with changing terminating decimal fraction.

**CORRECT ✓**

Let \( n = 0.151515\ldots \) Let \( n \) be the given number
\[ 100n = 15.1515\ldots \] Multiply both sides of the equation by 100

Subtracting \( n \) from 100\( n \) we have

\[
\begin{align*}
100n &= 15.1515\ldots \\
- n &= 0.151515\ldots \\
99n &= 15
\end{align*}
\]

\[ n = \frac{15}{99} = \frac{5}{33} \]

Therefore, \( 0.151515\ldots = \frac{5}{33} \)

- If the repetend consists of 3 digits, multiply both sides by 1000 and subtract \( n \) from 1000\( n \).

**Example** Convert 0.135135…

Let \( n = 0.135\ldots \) Let \( n \) be the given number
\[ 1000n = 135.135135\ldots \] Multiply both sides by 1000

Subtracting \( n \) from 1000\( n \), we have
1000n = 135.135...
\[ \begin{align*}
- n & = 0.135... \\
999n & = 135
\end{align*} \]
\[ n = \frac{135}{999} \]

Therefore, \( 0.135125... = \frac{135}{999} \)

Example 6 Convert 0.125... to fraction.

**COMMON ERROR !!!**

\( 0.125... = \frac{125}{1000} = \frac{1}{8} \)  
Converting repeating non terminating decimal to fraction is the same with converting terminating decimal to fraction

The error found in illustration above is the process of changing a non terminating decimal to fraction since it is not the same with changing terminating decimal fraction.

**CORRECT ✓**

Look at the process involved in converting repeating non-terminating decimal.

Let \( n = 0.125... \) Let \( n \) be the given number

\[ 1000n = 125.125125... \]  
Multiply both sides by 1000

Subtracting \( n \) from 1000\( n \), we have
\[1000n = 125.125\ldots\]
\[- n = 0.125\ldots\]
\[99n = 125\]

\[n = \frac{125}{999}\]

Therefore, \(0.125125\ldots = \frac{125}{999}\)
**Self – check 1.2**

**A. Change the given decimals to fractions**

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</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.4</td>
<td>5. 0.36</td>
</tr>
<tr>
<td>2.</td>
<td>0.55</td>
<td>6. 0.05</td>
</tr>
<tr>
<td>3.</td>
<td>0.87</td>
<td>7. 0.112</td>
</tr>
<tr>
<td>4.</td>
<td>0.125</td>
<td>8. 0.085</td>
</tr>
</tbody>
</table>

**B. Change the repeating, nonterminating decimals to fractions.**

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.44...</td>
<td>5. $\frac{13}{35}$</td>
</tr>
<tr>
<td>2.</td>
<td>0.2424...</td>
<td>6. 0.123123...</td>
</tr>
<tr>
<td>3.</td>
<td>$\frac{7}{5}$</td>
<td>7. 0.161616...</td>
</tr>
<tr>
<td>4.</td>
<td>$\frac{5}{5}$</td>
<td>8. 5. 252525...</td>
</tr>
</tbody>
</table>
Lesson 2  Rational Numbers on the Number Line

Objectives: At the end of the lesson, the student is expected to:

1. Use appropriate relation symbols in comparing fractions.
2. Determine the absolute value of the rational number.
3. Arrange fractions in ascending and descending order.

Concepts

Every rational number can be represented as a point on the number, since points on it can correspond to a part or a fraction of a whole.

Example 1

A. Observe the number line.

A point midway 1 and 2 represents the fraction \( \frac{3}{2} \) which is also 1 ½ or 1.5.

Also the point midway between – 1 and – 2 represents \( -\frac{3}{2} \) which is also – 1 ½ or 1.5.

B. In the same manner, a negative fraction can be located on the number line.
If the units are divided into fifths, $-\frac{3}{5}$ can be located to the left of 0, and $+\frac{3}{5}$ can be located at the right of 0.

In Example B, like integers, fractions and decimals also have opposites as indicated by $-\frac{3}{5}$ and $\frac{3}{5}$. They also belong to signed numbers, in this case.

All properties that are true with integers are also true with rational numbers.

The rational numbers on the number line increase from left to right. The relation symbols ($>$, $<$, $\geq$, $\leq$) are also used to form relationships between rational numbers.

**C. Relationships between the rational numbers.**

\[
\begin{array}{cccccccc}
-0.8 & -0.6 & -0.4 & -0.2 & 0.2 & 0.4 & 0.6 & 0.8 \\
-1 & -\frac{4}{5} & -\frac{3}{5} & -\frac{2}{5} & -\frac{1}{5} & 0 & \frac{1}{5} & \frac{2}{5} & \frac{3}{5} & \frac{4}{5} & 1 \\
\end{array}
\]

\[\frac{2}{5} < \frac{4}{5}\]

0.8 > 0.4
From our number line, notice that the numbers of intervals or the distance from zero is both positive and negative rational numbers is the same. This is clearly shown in the figure below.

The number of intervals or distance from zero of a positive number is equal to the number of intervals or distance of the corresponding negative number from zero. This is called the absolute value of the two numbers denoted by \( \lvert \cdot \rvert \) read as “the absolute value of.” The absolute value of 6 is 6 and the absolute value of \(-6\) is also 6. In symbols, we have

\[
\lvert 6 \rvert = 6 \text{ and } \lvert -6 \rvert = 6
\]

In general for any integer \( n \)
\[
\lvert n \rvert = n \text{ if } n > 0 \\
\lvert n \rvert = n \text{ if } n < 0 \\
\lvert n \rvert = 0 \text{ if } n = 0.
\]

The absolute value of \( \frac{+1}{5} \) or \( \left| \frac{+1}{5} \right| \) is \( \frac{1}{5} \).

The absolute value of \( \frac{-3}{4} \) or \( \left| \frac{-3}{4} \right| \) is \( \frac{3}{4} \).

The absolute value of \(-0.85\) or \( \left| -0.85 \right| \) is 0.85
The absolute value of 1.5 or $|1.5|$ is 1.5.

**Example 2.** Suppose you are asked to compare two fractions which are similar:

Look at the examples given below.

A. **Which is larger 8/9 or 7/9?**
   
   $8/9 > 7/9$ since $8 > 7$

B. **Which is larger 3/8 or 5/8?**
   
   $3/8 < 5/8$ since $3 < 5$

Can you formulate the rule in comparing similar fractions?

**Rules in Comparing Similar Fractions**

To compare similar fractions, just compare their numerators. The fraction with the bigger numerator is larger.

**Example 3.** How would you compare dissimilar fractions? If you little of this, your knowledge will be enhanced in the following section.

**Dissimilar fractions** are fractions with different denominators.

**Example**

$1/3$, $3/4$, $3/7$ and $3/5$ are dissimilar fractions since they have different denominators.

**Rule in Comparing Dissimilar Fractions with Equal Numerators**

To compare dissimilar fractions with equal numerators, the fraction with the smaller denominator is larger.
A. Which is greater $\frac{4}{5}$ or $\frac{4}{7}$?

**COMMON ERROR !!!**

$$\frac{4}{7} > \frac{4}{5} \quad \text{since } 7 > 5$$

The error in the illustrative example above is that the student thinks that the fraction with bigger denominator is the larger fraction.

**CORRECT ✓**

$$\frac{4}{5} > \frac{4}{7} \quad \text{since } 5 < 7$$

When comparing fractions with the same numerators, it is appropriate to compare the numerators not the denominators. The fraction with bigger numerator is the larger.

B. Which is larger $\frac{9}{4}$ or $\frac{9}{7}$?

**COMMON ERROR !!!**

$$\frac{9}{7} > \frac{9}{4} \quad \text{since } 7 > 4$$

The error in the illustrative example above is that the student thinks that the fraction with bigger denominator is the larger fraction.
When comparing fractions with the same numerators, it is appropriate to compare the numerators not the denominators. The fraction with bigger numerator is the larger.

Let us consider dissimilar fractions with unequal numerators. The following procedure will help you in comparing dissimilar fractions with unequal numerators.

The **Least Common Denominator (LCD)** is the **Least Common Multiple** of two or more denominators.

How to find the Least Common Denominator:

1. Find the Greatest Common Factor of the denominators.
2. Multiply the denominators together.
3. Divide the product of the denominators by the Greatest Common Factor.

**Example: Find the LCD of 2/9 and 3/12**

1. Determine the Greatest Common Factor of 9 and 12 which is 3
2. Either multiply the denominators and divide by the GCF (9*12=108, 108/3=36)
3. OR - Divide one of the denominators by the GCF and multiply the quotient times the other denominator (9/3=3, 3*12=36)
Comparing Dissimilar Fractions with Unequal Numerators

1. Get the LCD (Least Common Denominator), which is the LCM (Least Common Multiple) of the denominators.
2. Change all fractions to similar fractions using the LCD as the common denominator.
3. Compare the fractions following the rule in comparing similar fractions.

C. Which is greater 2/3 or 3/5?

COMMON ERROR !!!

\[
\frac{3}{5} > \frac{2}{3}
\]

Dissimilar fractions are compared using their denominators. The larger the denominator, the larger the fraction.

The error in this example is that the denominators of dissimilar fractions are compared; it is an error since we have to find first the LCD of the two fractions.

CORRECT √

First, we get the LCD. The LCD is 15.

Next, we change the fractions to similar fractions using the LCD as the common denominator.

\[
\frac{2}{3} = \frac{2}{3} \cdot \frac{5}{5} = \frac{10}{15}
\]

\[
\frac{3}{5} = \frac{3}{5} \cdot \frac{3}{3} = \frac{9}{15}
\]
Since $10/15 > 9/15$, then $2/3 > 3/5$.

In comparing dissimilar fractions, we have to find first the LCD of the two fractions and then compare the results using the rule of comparing similar fractions.

D. Which is greater $15/7$ or $14/6$?

**COMMON ERROR !!!**

\[
\frac{15}{7} > \frac{14}{6}
\]

The dissimilar fractions are compared using their numerators.

The error in this example is that the denominators of dissimilar fractions are compared; it is an error since we have to find first the LCD of the two fractions.

**CORRECT ✓**

The LCD of the given fractions is 42.

We change the fractions to similar fractions using 42 as the common denominator.

\[
\frac{15}{7} = \frac{15(6)}{7(6)} = \frac{90}{42}
\]

\[
\frac{14}{6} = \frac{14(7)}{6(7)} = \frac{98}{42}
\]

Since $90/42 < 98/42$, then $15/7 < 14/6$. 
In comparing dissimilar fractions, we have to find first the LCD of the two fractions and then compare the results using the rule of comparing similar fractions.

E. Arrange the fractions 4/5, 6/5, and 2/5 from smallest to largest.

Since the fractions are similar, the fraction with the smallest numerator has the least value.

Thus, the ascending order should be 2/5, 4/5, and 6/5.

F. Arrange 3/8, 5/6 and 2/3 in increasing order.

<table>
<thead>
<tr>
<th>COMMON ERRORS !!!</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \frac{2}{3} ), ( \frac{3}{8} ), ( \frac{5}{6} )</td>
</tr>
<tr>
<td>since ( 2 &lt; 3 ), then ( 3 &lt; 5 )</td>
</tr>
<tr>
<td>2. ( \frac{2}{3} ), ( \frac{5}{6} ), ( \frac{3}{8} )</td>
</tr>
<tr>
<td>since ( 3 &lt; 6 ) and ( 6 &lt; 8 )</td>
</tr>
</tbody>
</table>

The error in number (1) is that the dissimilar fractions are compared using the numerators and number (2) are compared using the denominators. The student failed to find the Least Common Denominator of the fractions.
Since the fractions are dissimilar, we change them first to similar fractions.

The LCD is 24. Thus,

\[
\frac{3}{8} = \frac{3(3)}{8(3)} = \frac{9}{24}
\]

\[
\frac{5}{6} = \frac{5(4)}{6(4)} = \frac{20}{24}
\]

\[
\frac{2}{3} = \frac{2(8)}{3(8)} = \frac{16}{24}
\]

Using the rule in ordering similar fractions, \(9/24 < 16/24 < 20/24\).

Thus, the ascending should be \(3/8, 2/3, \) and \(5/6\).

To order fractions with dissimilar fractions, **it is proper to find first the Least Common Denominator and compare them using the rule of similar fractions.**

**Points to Remember**

1. If fractions have the same denominator, the fraction with the bigger numerator is larger.
2. The principle of comparing dissimilar fractions is to change the given fractions to similar fractions.
Self-check 2

A. Fill in the spaces with the correct relation symbols.

1. \( \frac{6}{7} \underline{__} \frac{7}{8} \) \hspace{2cm} 5. \( \frac{2}{3} \underline{__} \frac{3}{5} \) \hspace{2cm} 9. \( \frac{3}{5} \underline{__} \frac{2}{3} \)

2. \( \frac{5}{4} \underline{__} \frac{6}{5} \) \hspace{2cm} 6. \( \frac{5}{7} \underline{__} \frac{15}{24} \) \hspace{2cm} 10. \( \frac{2}{6} \underline{__} \frac{3}{6} \)

3. \( \frac{4}{3} \underline{__} \frac{3}{2} \) \hspace{2cm} 7. \( \frac{15}{32} \underline{__} \frac{25}{64} \)

4. \( \frac{3}{5} \underline{__} \frac{3}{7} \) \hspace{2cm} 8. \( \frac{1}{3} \underline{__} \frac{5}{6} \)

B. Arrange each set of fractions from smallest to largest.

1. \( \frac{3}{4}, \frac{1}{2}, \frac{2}{5} \) \hspace{2cm} 5. \( \frac{2}{3}, \frac{5}{7}, \frac{5}{8} \) \hspace{2cm} 9. \( \frac{2}{3}, \frac{1}{2}, \frac{3}{8} \)

2. \( \frac{8}{9}, \frac{13}{9}, \frac{11}{9} \) \hspace{2cm} 6. \( \frac{1}{2}, \frac{3}{4}, \frac{1}{3} \) \hspace{2cm} 10. \( \frac{1}{2}, \frac{1}{6}, \frac{1}{3} \)

3. \( \frac{7}{8}, \frac{4}{5}, \frac{5}{6} \) \hspace{2cm} 7. \( \frac{2}{3}, \frac{3}{4}, \frac{5}{8} \)

4. \( \frac{5}{8}, \frac{1}{4}, \frac{1}{16}, \frac{11}{12} \) \hspace{2cm} 8. \( \frac{1}{2}, \frac{5}{6}, \frac{7}{8} \)
C. Determine the value of the following.

1. \( \left| -\frac{1}{2} \right| \)
2. \( \left| \frac{5}{11} \right| \)
3. \( |0| \)
4. \( |-4| \)
5. \( |8| \)
6. \( |-8 + 4| \)
7. \( \left| \frac{3}{4} \right| \)
8. \( |0 - 1| \)
9. \( \left| -\frac{2}{3} \right| \)
10. \( |5 - 1| \)
Lesson 3  Operations with Fractions

At the end of the lesson, the student is expected to:

1. perform operations on fractions;
2. solve problems involving fractions.

Concepts

All the rules which apply to operations on integers hold for operations with fractions. Using the number line, any movement from an initial point going to the right is positive, and any movement going to the left is negative.
3.1 Addition and Subtraction of Fractions

Like whole numbers, fractions can also be added and subtracted. Study the following examples that illustrate how to add and subtract fractions and also its rules.

- If $a$, $b$, and $c$ denote integers, and if $b \neq 0$, then \( \frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \).

- To add fractions with different denominators, convert the fractions to equivalent forms with the same denominator. This requires looking for the least common denominator (LCD) of the fractions.

- If $a$, $b$, and $c$ denote integers, and if $b \neq 0$, then \( \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b} \).

- To subtract fractions with different denominators, convert the fractions to equivalent forms with the same denominator. This requires looking for the least common denominator (LCD) of the fractions.

Example 1 Finding the sum.

\[
\frac{2}{9} + \frac{5}{9}
\]

COMMON ERROR !!!

\[
\frac{2}{9} + \frac{5}{9} = \frac{2 + 5}{9 + 9} = \frac{7}{18}
\]

Fractions' numerators and denominators are treated as separate whole numbers.
Adding the numerators and denominators of two fractions is inappropriate. Students fail to recognize that denominators define the size of the fractional part and that numerators represent the number of this part.

**CORRECT √**

\[
\frac{2}{9} + \frac{5}{9} = \frac{2+5}{9} = \frac{7}{9}
\]

In adding fractions with common denominators, it is appropriate to add their numerators and copy the common denominator then reduce to the lowest term if possible.

**Example 2** Finding the sum.

\[
\left(\frac{-3}{7}\right) + \left(\frac{-5}{7}\right)
\]

**COMMON ERRORS !!!**

1. \(\left(\frac{-3}{7}\right) + \left(\frac{-5}{7}\right) = \frac{-2}{7}\) Subtracting 3 from 5. It is an error.

2. \(\left(\frac{-3}{7}\right) + \left(\frac{-5}{7}\right) = \frac{(-3) + (-5)}{7 + 7} = \frac{-8}{14}\) Fractions' numerators and denominators are treated as separate whole numbers. It is an error.

In number (1), it is inappropriate to subtract 3 from 5 since they have the same signs. Students fail to recognize the rules in
adding integers with the same signs. In number (2), Adding the numerators and denominators of two fractions is inappropriate. Students fail also to recognize that denominators define the size of the fractional part and that numerators represent the number of this part.

**CORRECT √**

\[
\left( -\frac{3}{7} \right) + \left( -\frac{5}{7} \right) = \left( -3 \times \frac{1}{7} \right) + \left( -5 \times \frac{1}{7} \right) \quad \text{Changing } \frac{-3}{7} \text{ to } -3 \times \frac{1}{7} \text{ and } \frac{-5}{7} \text{ to } -5 \times \frac{1}{7}
\]

\[
= [(-3) + (-5)] \times \frac{1}{7} \quad \text{Distributive property}
\]

\[
= (-8) \times \frac{1}{7} \quad \text{Simplifying}
\]

\[
= \frac{-8}{7} \text{ or } -1 \frac{1}{7}
\]

In adding fractions with negative numerators, it is proper to add their numerators and copy the common sign, then copy the common denominator. Reduce the result if possible.

**Example 3 Finding the sum.**

\[
\frac{4}{3} + \frac{5}{3}
\]
COMMON ERRORS !!!

1. \( \frac{4}{3} + \frac{5}{3} = \frac{9}{3} \)  
   It is not simplified. It is an error.

2. \( \frac{4}{3} + \frac{5}{3} = \frac{4 + 5}{3 + 3} = \frac{9}{6} \)
   Fractions’ numerators and denominators are treated as whole numbers. It is an error.

In number (1), it is not appropriate to just add the numerators and copy the common denominators, simplify the result if possible. In number (2), adding the numerators and denominators of two fractions is inappropriate. Students fail also to recognize that denominators define the size of the fractional part and that numerators represent the number of this part.

CORRECT √

\[
\frac{4}{3} + \frac{5}{3} = \left(4 \times \frac{1}{3}\right) + \left(5 \times \frac{1}{3}\right) \quad \text{Changing } \frac{4}{3} \text{ to } 4 \times \frac{1}{3} \text{ and } \frac{5}{3} \text{ to } 5 \times \frac{1}{3} \\
= (4 + 5) \times \frac{1}{3} \quad \text{Distributive property} \\
= 9 \times \frac{1}{3} \quad \text{Simplifying} \\
= 3
\]

In adding improper fractions with the same denominators, it is correct to rename it into a whole number and a proper fraction, then add their whole number parts and multiply the result to the common proper fraction. Simplify afterwards.
Example 4  

Finding the sum.

\[ \frac{4}{5} + \frac{4}{10} \]

**COMMON ERROR !!!**

\[ \frac{4}{5} + \frac{4}{10} = \frac{8}{10} \]

Failing to find common denominator when adding unlike denominators is an error.

In the example above, it is inappropriate to simply add the numerators and copy the bigger denominator since they have different denominators. Students failed to find the Least Common Denominator (LCD) between the two fractions.

**CORRECT √**

\[ \frac{4}{5} + \frac{4}{10} = \frac{4(2)}{5(2)} + \frac{4}{10} \]

Get the LCD. The LCD is 10

\[ = \frac{8}{10} + \frac{4}{10} \]

\[ = \left( 8 \times \frac{1}{10} \right) + \left( 4 \times \frac{1}{10} \right) \]

Changing \( \frac{8}{10} \) to \( 8 \times \frac{1}{10} \) and \( \frac{4}{10} \) to \( 4 \times \frac{1}{10} \)

\[ = (8 + 4) \times \frac{1}{10} \]

Distributive Property

\[ = \frac{12}{10} \]

Simplifying
It is appropriate to find first the Least Common Denominator to have similar fractions. Add the fractions following the rules in adding similar fractions, and then reduce the result to the simplest form.

**Example 5**

Finding the sum.

\[ \frac{3}{5} + \frac{2}{3} \]

**COMMON ERROR !!!**

Adding the denominators in dissimilar fractions and crossing out common numbers from the numerator and denominator are errors.

One of the common errors in adding dissimilar fractions is cancelling out the numbers which are common in the numerator and denominator. It is inappropriate since the numbers in the numerator is separated by an addition sign as well as in the denominator.
The most appropriate way of adding dissimilar fractions is to find the Least Common Denominator to change it to similar fractions. Add them following the rules of adding similar fractions and simplify.

**Example 6**  
Consider this situation.

Mrs. Cruz needs 2 ¼ cups of sifted flour to make a plain cake and 1 ½ cups for a pineapple sponge cake. Find the total amount of flour she needs.
a. **Understanding the problem**

Find the total amount of flour Mrs. Cruz needs.

- 2 ¼ cups of sifted flour for plain cake
- 1 ½ cups for a pineapple sponge cake

b. **Operations used**

Plan what operation to be used: **Addition**

c. **Set the equation:**

\[ 2 \frac{1}{4} + 1 \frac{1}{2} = N \]

d. **Finding the answer:**

- \[ 2 \frac{1}{4} + 1 \frac{1}{2} = \left( 2 + \frac{1}{4} \right) + \left( 1 + \frac{1}{2} \right) \] Changing mixed numbers

- \[ (2 + 1) + \left( \frac{1}{4} + \frac{1}{2} \right) \] Associative property

- \[ 3 + \left( \frac{1}{4} + \frac{2}{4} \right) \] Changing to similar fractions

- \[ 3 + \frac{3}{4} \] Simplifying

- \[ 3 \frac{3}{4} \]

Thus, Mrs. Cruz needs 3 ¾ cups of flour.

**Example 7** Finding the difference.

\[
\frac{4}{7} - \frac{3}{7}
\]
Subtracting the numerators and denominators of two fractions is inappropriate. Students fail to recognize that denominators define the size of the fractional part and that numerators represent the number of this part.

In subtracting fractions with common denominators, it is appropriate to subtract their numerators and copy the common denominator then reduce to the lowest term if possible.

Example 8 Finding the difference.

\[ 5 \frac{2}{3} - 3 \frac{1}{2} \]
In subtracting mixed numbers, it is incorrect to subtract the numerators and denominators of the proper fractions. Failing to find the common denominator is also the error.

**CORRECT √**

\[
5 \frac{2}{3} - 3 \frac{1}{2} = \left( 5 + \frac{2}{3} \right) - \left( 2 + \frac{1}{2} \right) \quad \text{Changing mixed number}
\]

\[
= (5 - 2) + \left( \frac{2}{3} - \frac{1}{2} \right) \quad \text{Associative property}
\]

\[
= 3 + \left( \frac{4}{6} - \frac{3}{6} \right) \quad \text{Changing to similar fractions}
\]

\[
= 3 + \frac{1}{6} \quad \text{Simplifying}
\]

\[
= 3 \frac{1}{6}
\]

In subtracting mixed numbers, rename the dissimilar proper fractions into similar fractions, and then subtract the whole number parts and the similar fractions. Reduce the result if possible.
Example 9 Finding the difference.

\[ 25 \frac{1}{7} - 4 \frac{3}{7} \]

**COMMON ERROR !!!**

\[ 25 \frac{1}{7} - 4 \frac{3}{7} = (25 - 4) + \frac{1}{7} - \frac{3}{7} \]

= 11 + \left( \frac{1-3}{7} \right) \quad \text{Subtracting directly the smaller proper fraction in a mixed number at the minuend to the bigger proper fraction in a mixed number at the subtrahend is an error.}

= 11 \frac{2}{7}

In the example above, it is inappropriate to subtract directly the smaller proper fraction in a mixed number at the minuend to the bigger proper fraction in a mixed number at the subtrahend. Students fail to borrow one part from the whole number part.

**CORRECT ✓**

\[ 25 \frac{1}{7} - 4 \frac{3}{7} = 24 + \left( \frac{7}{7} + \frac{1}{7} \right) - 4 \frac{3}{7} \quad \text{Borrowing 1 or} \quad \frac{7}{7} \quad \text{from 25} \]

= 24 + \frac{8}{7} - 4 + \frac{3}{7} \quad \text{Changing mixed number}

= (24 - 4) + \left( \frac{8}{7} - \frac{3}{7} \right) \quad \text{Associative property}

= 20 + \frac{5}{7} \quad \text{Simplifying}

= 20 \frac{5}{7}
When subtracting mixed numbers, it is appropriate to borrow one part from the whole part if the proper fraction of the first mixed number is less than the proper fraction of the second mixed number.

Example 10

Joey has 5 \( \frac{3}{4} \) m of dress material which he bought in a department store. He needs only 3 \( \frac{1}{2} \) for his dress. How many meters of it will be left for his younger brother’s use?

a. Understanding the problem
How many meters of the dress material will be left for his younger brother’s use?

5 \( \frac{3}{4} \) m dress material
3 \( \frac{1}{2} \) m for Joey’s dress

b. Operations used
Plan what operation to be used: **Subtraction**

c. Set the equation:

\[ 5 \frac{3}{4} - 3 \frac{1}{2} = N \]

d. Finding the answer:

- \( 5 \frac{3}{4} - 3 \frac{1}{2} = (5 + \frac{3}{4} ) - (3 + \frac{1}{2}) \) Changing mixed numbers

- \( (5 - 3) + (\frac{3}{4} - \frac{1}{2}) \) Associative property

- \( 2 + (\frac{3}{4} - \frac{2}{4}) \) Changing to similar fractions
\[ \cdot \quad 2 + \frac{1}{4} \quad \text{Simplifying} \]

\[ \cdot \quad 2 \frac{1}{4} \]

Thus, 2 \( \frac{1}{4} \) m of material will be left for his brother’s use.
Self-check 3.1

A. Find the sum. Express the answers in lowest term.

1. $\frac{7}{11} + \frac{3}{11}$
2. $\frac{-2}{6} + \frac{5}{12} + \frac{3}{12}$
3. $3\frac{2}{9} + 4\frac{2}{3}$
4. $\frac{-3}{8} + \frac{-3}{4}$
5. $\frac{3}{5} + \frac{3}{10}$
6. $\frac{2}{8} + \frac{7}{8}$
7. $3\frac{1}{4} + 5\frac{2}{4}$
8. $\frac{1}{2} + \frac{2}{5} + \frac{7}{9}$
9. $\frac{4}{5} + \frac{3}{7}$
10. $\frac{3}{2} + \frac{5}{8}$

B. Find the difference. Express the answers in lowest term.

1. $\frac{5}{6} - \frac{1}{3}$
2. $\frac{7}{8} - \frac{5}{6} - \frac{3}{4}$
3. $\frac{6}{7} - \frac{3}{5}$
4. $\frac{9}{16} - \frac{5}{16}$
5. $6\frac{3}{4} - 3\frac{1}{3}$
6. $12 - 6\frac{3}{4}$
7. $\frac{4}{5} - \frac{2}{3}$
8. $8\frac{3}{10} - 1\frac{5}{6}$
9. $9\frac{1}{8} - 4\frac{3}{8}$
10. $\frac{3}{4} - \frac{2}{3}$
C. Solve the following problems.

1. Lino used $\frac{1}{2}$ bar of butter for his hot cake and $\frac{1}{3}$ bar of butter for the toasted bread of the father. How much butter did he use in all?

2. A 4 $\frac{1}{4}$ lb chicken weighed 3 $\frac{1}{8}$ lb when dressed. Find the loss in weight.

3. Mrs. Santillan bought $6 \frac{3}{9}$ kilos of beef, $8 \frac{1}{2}$ kilos of bangus, and $14 \frac{3}{4}$ kilos of chicken. How many kilos of meat did she buy?

4. Mr. Jack, a tailor cut $6 \frac{1}{2}$ meters of cloth from a 16 meter cloth. How many meters remained?

5. Karl needs $5 \frac{3}{4}$ meters of plastic cover for his notebooks, $8 \frac{2}{3}$ meters for the books, and $3 \frac{2}{5}$ meters for project folders. How many meters of plastic cover does he need in all?

6. Mr. Benedict donated $9 \frac{1}{2}$ hectares of his $14 \frac{3}{4}$ hectares of land. How many hectares of land did he have left?

7. JRB Company spends $\frac{1}{9}$ of the capital in advertisement, $\frac{2}{5}$ for the cost of goods and $\frac{1}{8}$ for maintenance. What fractional part of their capital is allotted for other needs?
8. Find the total weight in grams of a can and its content if the can weighs \( \frac{5}{8} \) gram and its contents weigh \( 19 \frac{11}{12} \) grams.

9. Mayor XYZ’s staff bought sack of rice containing the following weights to be given to the typhoon victims: \( 52 \frac{3}{4} \) kg, \( 103 \frac{2}{3} \) kg, and \( 203 \frac{5}{24} \) kg. Find the total weight of the rice.

10. A container was \( \frac{7}{9} \) full of oil. Enough oil was added to fill the container to \( \frac{11}{12} \) full. How much oil was added?
3.2 Multiplication and Division of Fractions

Exploration

You learned how to add and subtract fractions from the previous lesson. This time, let us consider multiplication and division of fractions. Study the following rule in multiplying and dividing fractions.

Example 1

Multiply: \( \frac{2}{3} \times \frac{1}{3} \)

COMMON ERROR !!!

\[
\frac{2}{3} \times \frac{1}{3} = \frac{2}{3} \quad \text{Leaving the denominator unchanged in fraction addition and multiplication problem is an error.}
\]

In the example above, the error is the copying of the common denominator in the answer since in multiplying fractions the rule is multiply the numerator and also the denominator. Simplify the result if possible.

CORRECT √

\[
\left( \frac{2}{3} \right) \times \left( \frac{1}{3} \right) = \left( 2 \times \frac{1}{3} \right) \times \left( 1 \times \frac{1}{3} \right) \quad \text{Changing } \frac{2}{3} \text{ to } 2 \times 1/3 \text{ and } \frac{1}{3} \text{ to } 1 \times 1/3
\]

\[
= (2 \times 1) \times \left( \frac{1}{3} \times \frac{1}{3} \right) \quad \text{Commutative and associative properties of multiplication}
\]
Simplifying

It is appropriate when multiplying fractions, to rename both fractions into a whole number and proper fractions and then multiply and reduce the result to the lowest term if possible.

Example 2

Multiply: \(2 \frac{1}{2} \times 3 \frac{1}{3}\)

\[
2 \frac{1}{2} \times 3 \frac{1}{3} = 2 \times 3 + \frac{1}{2} \times \frac{1}{3}
\]

\[
= 6 + \frac{1}{6}
\]

\[
= 6 \frac{1}{6}
\]

In the example above, it is an error since whole numbers and the fractions are multiplied separately. Students failed to recognize that whole numbers and fractions are supposed to be multiplied.
**CORRECT √**

\[
2\frac{1}{2} \times 3\frac{1}{3} = \frac{5}{2} \times \frac{10}{3} \quad \text{Changing mixed number to improper fraction.}
\]

\[
= \left( 5 \times \frac{1}{2} \right) \times \left( 10 \frac{1}{3} \right) \quad \text{Changing} \ \frac{5}{2} \ \text{to} \ 5 \times \frac{1}{2} \ \text{and} \ \frac{10}{3} \ \text{to} \ 10 \times \frac{1}{3}
\]

\[
= (5 \times 10) \times \left( \frac{1}{2} \times \frac{1}{3} \right) \quad \text{Commutative and associative properties of multiplication}
\]

\[
= 50 \times \frac{1}{6} \quad \text{Simplifying}
\]

\[
= \frac{50}{6}
\]

\[
= 8\frac{2}{6} \text{ or } 8\frac{1}{3} \quad \text{Renaming improper fraction to mixed number and reducing to lowest term.}
\]

In multiplying mixed numbers, it is appropriate to change it to improper fractions, multiply the whole numbers part and also the fractional part and then multiply the result of each part. Reduce the result if possible.

**Example 3**  
Consider this situation.

It takes Karen 1 \(\frac{1}{4}\) hours to complete a piece of cross-stitch. How many hours will it take her to complete a dozen pieces?
Changing the mixed number to improper fraction

\[
1 \frac{1}{4} \times 12 = \frac{5}{4} \times 12
\]

Changing \(5 \frac{1}{4}\) to \(5 \times \frac{1}{4}\)

\[
= (5 \times \frac{1}{4}) \times 12
\]

Commutative and associative properties of multiplication

\[
= (5 \times 12) \times \frac{1}{4}
\]

Simplifying

\[
= 60 \times \frac{1}{4}
\]

\[
= \frac{60}{4} \text{ or } 15
\]

Thus, Karen can complete a dozen pieces in 15 hours.

Example 4

Divide: \(\frac{5}{7} \div \frac{3}{4}\)

COMMON ERROR !!!

\[
\frac{5}{7} \div \frac{3}{4} = \frac{25}{21}
\]

Failing to understand the invert-and-multiply procedure for solving fraction division problems

In the given example above, the error of the student is its failure to understand the invert-and-multiply procedure for solving fraction division problems. The student did not write the reciprocal or the multiplicative inverse of the divisor.
CORRECT √

\[
\frac{5}{7} \div \frac{3}{4} = \frac{5}{7} \times \frac{4}{3}
\]

\[
= \frac{20}{21}
\]

The appropriate way of solving division problems is to find the multiplicative inverse or reciprocal of the divisor and then follow the process of multiplying rational numbers.

Example 5 Finding the quotient.

\[
\left(5 \frac{1}{2}\right) \div \left(2 \frac{1}{3}\right)
\]

COMMON ERROR !!!

\[
5 \frac{1}{2} \div 2 \frac{1}{3} = \frac{5}{2} \times \frac{2}{3} = \frac{10}{6}
\]

Multiplying the whole number to its proper fraction is an error.

\[
= 1 \frac{4}{6} \text{ or } 1 \frac{2}{3}
\]

The error in this example is the student multiplies the whole number to its proper fraction. It's an error since it violates the rule of renaming mixed number to an improper fraction.
The proper way of dividing mixed numbers is to change the mixed numbers into improper fractions and then find the multiplicative inverse of the divisor and follow the process of multiplying rational numbers. Change the answer (if is an improper fraction) to a mixed number and reduce if possible.

**Example 6**  
**Finding the quotient**

\[
\frac{5}{0}
\]

**COMMON ERROR !!!**

\[
\frac{5}{0} = 0 \quad \text{Dividing a whole number and zero is zero}
\]
The illustrated example above has an error since division of a number and zero is undefined. The answer of a number and zero cannot be equal to zero.

**CORRECT √**

\[
\frac{5}{0}, \text{undefined} \quad \text{Division by zero is not allowed}
\]

The illustration above is correct since division of a number and zero is not allowed and it is labeled as undefined.

**Example 7**

A 25 – liter tank to be filled with water by repeatedly pouring from a can which holds 2½ liters. How many pourings are needed to fill the tank?

a. **Understanding the problem**
   How many pourings are needed to fill the tank?
   20 – Liter tank
   2 ½ liter can

b. **Operations to be used:**
   Plan what operation to be used: **Division**

c. **Set the equation:**
   \[25 ÷ 2 \frac{1}{2} = N\]

d. **Finding the answer:**
   - \[25 ÷ 2 \frac{1}{2} = 20 ÷ \frac{5}{2}\] Changing the mixed number
   - \[\frac{25}{1} \times \frac{2}{5}\] Multiplicative Inverse property
Thus, it requires 10 pourings of water to fill the 25 – liter tank.
Self-check 3.2

A. Find each product. Express all answers in lowest term.

1. $\frac{9}{15} \times 3$  
2. $1\frac{3}{4} \times \frac{18}{15}$  
3. $2\frac{1}{2} \times 1\frac{2}{5} \times \frac{5}{4}$  
4. $\frac{5}{8} \times 28$  
5. $\frac{6}{12} \times \frac{8}{9}$  
6. $\frac{3}{8} \times \frac{5}{11}$  
7. $\frac{5}{9} \times \frac{4}{7} \times \frac{6}{8}$  
8. $2\frac{3}{5} \times 85$  
9. $2\frac{1}{3} \times 3\frac{1}{2}$

B. Find the quotient. Reduce the answer to lowest term.

1. $\frac{3}{10} \div \frac{3}{12}$  
2. $5\frac{1}{2} \div 2\frac{1}{3}$  
3. $21 \div 3\frac{1}{4}$  
4. $\frac{7}{16} \div \frac{3}{4}$  
5. $\frac{3}{4} \div \frac{2}{3}$  
6. $2\frac{1}{3} \div 3\frac{1}{2}$  
7. $4\frac{1}{3} \div \frac{2}{5}$  
8. $\frac{12}{9} \div 4$  
9. $\frac{15}{0}$

10. $\frac{3}{2} \div \frac{2}{3}$

C. Solve the following problems.

1. Mother bought 7 $\frac{1}{2}$ bags of sugar. Each bag weighed 1 $\frac{1}{4}$ kg. What is the total weight of the bags of sugar mother bought?

2. Gina has 12 $\frac{1}{2}$ meters of ribbon. She needs 1 $\frac{1}{4}$ meters of ribbon for each Christmas tree. How many
ribbons of this type can she make?
3. Johnny has a sack of fertilizer weighing 60 kilograms. He has to use \( \frac{1}{3} \) kilogram of fertilizer for 5 fruit trees in his farm. How many trees will be fertilized per sack?

4. Samuel has 20 piglets and each piglet consumes \( \frac{1}{4} \) kilo of feeds per day. How many kilos of feeds will the piglets consume in half a month?

5. A piece of wood 202 \( \frac{1}{2} \) inches long must be cut into pieces of \( 3 \frac{3}{4} \) inches each. How many pieces can be cut from the wood?

6. Ariel buys \( 3 \frac{3}{4} \) meters of material at PhP72 per meter. How much change does he get from a PhP500 bill?

7. How many pieces of \( 5 \frac{3}{8} \) centimeters wire can be cut from \( 32 \frac{1}{4} \) centimeters of wire?

8. Find the retail cost of \( 4 \frac{3}{8} \) meters of cotton at PhP15.85 per meter.

9. If a fly wheel makes six revolutions in \( \frac{5}{8} \) of a second, how many revolutions will it make in 80 seconds?
10. A company expects to raise PhP65 million in its current drive. The chairman projects that \( \frac{3}{5} \) of the funds will be raised on the first 12 weeks. What amount is expected to be raised on the first 12 weeks?
Bibliography


## Answer Key (Module)

### Self-check 1.1  page 10

**A.**

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### Self-check 1.2  page 18

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B.

1. \( \frac{4}{9} \) \hspace{2cm} 6. \( \frac{41}{333} \)
2. \( \frac{24}{99} \) \hspace{2cm} 7. \( \frac{16}{99} \)
3. \( \frac{75}{99} \) \hspace{2cm} 8. \( \frac{25}{99} \)
4. \( \frac{5}{99} \) \hspace{2cm} 9. \( \frac{28}{99} \)
5. \( \frac{135}{999} \) \hspace{2cm} 10. \( \frac{1}{3} \)

Self-check 2 pages 29 – 30

A.

1. < \hspace{2cm} 6. >
2. > \hspace{2cm} 7. >
3. < \hspace{2cm} 8. <
4. > \hspace{2cm} 9. <
5. > \hspace{2cm} 10. <

B.

1. \( \frac{2}{5}, \frac{1}{2}, \frac{3}{4} \) \hspace{2cm} 6. \( \frac{1}{3}, \frac{1}{2}, \frac{3}{4} \)
2. \( \frac{8}{9}, \frac{11}{9}, \frac{13}{9} \) \hspace{2cm} 7. \( \frac{5}{8}, \frac{2}{3}, \frac{3}{4} \)
3. \( \frac{4}{5}, \frac{5}{6}, \frac{7}{8} \) \hspace{2cm} 8. \( \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, \frac{7}{8} \)
4. \(\frac{1}{16}, \frac{1}{4}, \frac{5}{8}, \frac{11}{12}\)  
9. \(\frac{3}{8}, \frac{1}{2}, \frac{2}{3}\)  
5. \(\frac{5}{8}, \frac{2}{3}, \frac{5}{7}\)  
10. \(\frac{1}{6}, \frac{1}{3}, \frac{1}{2}\)

C.

1. \(\frac{1}{2}\)  
6. 4
2. \(\frac{5}{11}\)  
7. \(\frac{3}{4}\)
3. 0  
8. 1
4. 4  
9. \(\frac{2}{3}\)
5. 8  
10. 4

Self-check 3.1 pages 45 – 47

A.

1. \(\frac{10}{11}\)  
6. \(1\frac{1}{8}\)
2. \(\frac{1}{3}\)  
7. \(8\frac{3}{4}\)
3. \(7\frac{8}{9}\)  
8. \(1\frac{61}{90}\)
4. \(-1\frac{1}{8}\)  
9. \(1\frac{8}{35}\)
5. \(\frac{9}{10}\)  
10. \(4\frac{1}{8}\)
B.

1. $\frac{1}{2}$
2. $-\frac{17}{24}$
3. $\frac{9}{35}$
4. $\frac{1}{4}$
5. $3\frac{5}{12}$
6. $5\frac{1}{4}$
7. $\frac{2}{15}$
8. $6\frac{7}{15}$
9. $4\frac{3}{4}$
10. $\frac{5}{36}$

C.

1. $\frac{5}{6}$
2. $1\frac{1}{8}$
3. $29\frac{7}{12}$
4. $9\frac{1}{2}$
5. $17\frac{49}{60}$
6. $5\frac{1}{4}$
7. $\frac{131}{160}$
8. $20\frac{13}{24}$
9. $359\frac{5}{8}$
10. $\frac{5}{36}$
Self-check 3.2  pages 56 – 58

A.

1. $1 \frac{4}{5}$  
   6. $\frac{15}{88}$

2. $2 \frac{1}{10}$  
   7. $\frac{5}{21}$

3. $4 \frac{3}{8}$  
   8. 221

4. $17 \frac{1}{2}$  
   9. $8 \frac{1}{6}$

5. $\frac{4}{9}$  
   10. 1

B.

1. $1 \frac{1}{5}$  
   6. $\frac{2}{3}$

2. $2 \frac{5}{14}$  
   7. $10 \frac{5}{6}$

3. $6 \frac{6}{13}$  
   8. $\frac{1}{3}$

4. $\frac{7}{12}$  
   9. Undefined

5. $1 \frac{1}{8}$  
   10. $\frac{20}{21}$
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Note: This module is an output of my unpublished and unfinished master's degree thesis entitled “EFFECTIVENESS OF MODULAR INSTRUCTION IN ENHANCING STUDENTS’ FRACTION PROFICIENCY IN SECONDARY SCHOOLS OF ISLAND GARDEN CITY OF SAMAL”.