VaR Introduction I: Parametric VaR
Parametric VaR

Summary

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Value at Risk (VaR) Definition

◆ The maximum likely loss on a portfolio for a given probability defined as $x\%$ confidence level over $N$ days

◆ $\Pr(\text{Loss} > \text{VaR}(x\%)) < 1 - x\%$
Parametric VaR

VaR Roles

- Risk measurement
- Risk management
- Risk control
- Financial reporting
- Regulatory and economic capital
Parametric VaR

VaR Pros & Cons

◆ Pros
  ◆ Regulatory measurement for market risk
  ◆ Objective assessment
  ◆ Intuition and clear interpretation
  ◆ Consistent and flexible measurement

◆ Cons
  ◆ Doesn’t measure risk beyond the confidence level: tail risk
  ◆ Non sub-additive

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Three VaR Approaches

- Parametric VaR
- Historical VaR
- Monte Carlo VaR

The presentation focuses on parametric VaR.
Parametric VaR

Assumption
Asset returns follow normal distribution

Pros
Fast and simple calculation
Intuitive

Cons
Poor accuracy for non-linear products
Second order approximation
Hard to incorporate stress test
Parametric VaR Methodology

- Assuming an asset return/value change follows normal distribution, the quantile of 99% confidence level is $2.326 \sigma$ where $\sigma$ is standard derivation.

- If absolute return $X_1 - X_0$ is normally distributed, the 99% worse change of $X$ is $X_1 - X_0 = 2.326 \sigma$.

- The VaR is given by $\frac{\partial F}{\partial X} \Delta X = \frac{\partial F}{\partial X} \times 2.326 \times \sigma$ where $\frac{\partial F}{\partial X}$ is the delta.

- Similarly for a relative return $\frac{X_1 - X_0}{X_0}$, the VaR can be expressed as

$$\text{VaR} = \frac{\partial F}{\partial X} \Delta X = \frac{\partial F}{\partial X} \left( X_1 - X_0 \right) = \frac{\partial F}{\partial X} \times X_0 \times 2.326 \sigma$$
Parametric VaR Implementation

- For each asset/instrument/riskFactor, calibrate volatility $\sigma_i$ based on daily return
- For each risk factor pair, calibrate correlation $\rho_{ij}$
- Calculate the variance of a portfolio value change

$$V_p^2 = [\Delta(P_1)\sigma_1 \ldots \Delta(P_n)\sigma_n] \begin{bmatrix} \rho_{11} & \ldots & \rho_{1n} \\ \vdots & \ddots & \vdots \\ \rho_{n1} & \ldots & \rho_{nn} \end{bmatrix} [\Delta(P_1)\sigma_1 \ldots \Delta(P_n)\sigma_n]$$

- The portfolio VaR is $2.326 \sqrt{V_p^2}$
Parametric VaR

VaR Scaling

- Normally firms compute 1-day 99% VaR
- Regulators require 10-day 99% VaR
- Under IID assumption, 10-day VaR = $\sqrt{10} \times VaR_{1\text{-day}}$

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The only way to verify a VaR system is to backtest.

At a certain day, compute hypothetic P&L. If (hypothetic P&L > VaR) ➔ breach, otherwise, ok.

Hypothetic P&L is computed by holding valuation date and portfolio unchanged.

In one year period,

- If number of breaches is 0-4, the VaR system is in Green zone.
- If number of breaches is 5-9, the VaR system is in Yellow zone.
- If number of breaches is 10 or more, the VaR system is in Red zone.
Thanks!

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