The consort fantasies of William Byrd: The application of a new quantitative technique to describe \textit{fuga} subject deformation

Eamonn Bell
Trinity College, Dublin

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ABSTRACT

“The consort fantasias of William Byrd: The application of a new quantitative technique to describe fuga subject deformation”

Author: Eamonn Bell [09388826], SS Music and Mathematics
Supervisor: Andrew Johnstone (Department of Music, Trinity College, Dublin)

The polyphonic music of William Byrd (c.1543–1623) poses significant challenges to analysts of early music. The category of ‘imitative’ polyphony, which suggests a mode of analysis that seeks to identify successive identical (or at least, very similar) entries of a clearly-defined subject, is ill-fitting. Byrd’s polyphony is varied and discursive. On occasion, no two entries in his polyphonic passages (or ‘points’) are identical, either rhythmically or melodically. Recent studies of Byrd and continental contemporaries have offered the term fuga to describe this flexible formal procedure. John Milsom has provided a lexicon for the analysis of fuga, which accounts for the variation processes which the fuga subject undergoes. These processes are referred to here as subject deformations.

A brief summary of the analytical history of Byrd’s counterpoint in general, and of his consort music is provided. The most useful analytical terminologies and taxonomies are adopted for use in a quantitative model of fuga which tracks the amount subject deformation as it appears in successively deformed entries in Byrd’s points. A simple ‘subject deformation metric’ between pairs of entries is formally defined, and is used to describe both the amount and rate of deformation in Byrd’s points. Some improvements and graphical applications of the metric are described, and three model analyses of three- and four- part instrumental consort music are provided, to demonstrate the application of this new metric in the analysis of the music of Byrd.
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Introduction

The listening experience of Byrd’s fantasias for instrumental consort is the experience of a select handful of deftly-crafted tunes being cast, turn after turn, in a bewildering and fascinating series of combinations and variations. This is Byrd’s *fuga* in action. Thomas Morley, Byrd’s onetime student, unambiguously held that the genre of the Fantasy was the ‘principall and chiefest kind of musicke which is made without a dittie’¹ Byrd’s consort fantasias offer us an insight into a creative mind unfettered by extra-musical considerations, constructing polyphonic edifices according to the principles of his counterpoint. Byrd’s musical mind has been of considerable scholarly interest. His music, seeming to defy the straightened analytical categories that apply so readily to some of his near-contemporaries, has warranted several examinations and re-examinations in modern times. Perhaps only now are signs of a coherent system for describing his counterpoint emerging. This study aims to show that quantitative methods have an important role to play in forming and testing music-analytic hypotheses about early polyphony, and in particular about that of Byrd in light of the most recent successes in the analysis of his music.

Chapter 1 traces the history of modern Byrd analysis from Andrews’s first attempts in the 1940s, through the scant writings on the consort music, to Julian Grimshaw’s

most recent work in 2009 on Byrd’s early motets. The most useful and internally con-
sistent analytical taxonomies—those that do not seek to render deviant Byrd’s manner
of reiterating subjects in infinitely varied ways—are described and defined for later
application, and as guides for designing a quantitative measure of exactly those vari-
tions. Chapter 2 takes some of those analytical taxonomies and formally defines their
quantitative analog in the idea of a ‘subject deformation metric’ which tracks the level
of variation (or ‘deformation’) of the entries of Byrd’s contrapuntal passages as they
progress. This metric is used to determine heuristically the least deformed entries on
average, which suggests a candidate or candidates for the so-called ‘plain form’ of the
subject, without the need to posit its existence a priori. Finally, Chapter 3 brings the
new analytical apparatus to bear on a selection of Byrd’s three- and four- part fantasias
to illuminate their construction with the benefits of recent work on Byrd’s counterpoint
(Chapter 1), and in so doing, demonstrate possible applications of this new quantitative
analytical strategy (Chapter 2). The three ‘model analyses’ show that correct precondi-
tions for analysis and modern tools for the computational analysis of music, when taken
together, can provide musically appropriate insights into the intricate details of Byrd’s
counterpoint.
Chapter 1

Byrd’s consort music as viewed through the lens of present-day analysis

1.1 Terminology and Byrd’s counterpoint

William Byrd’s counterpoint is his varied and compelling musical argumentation. It is the way he treats musical primitives in combination. It is the setting of sections, subjects and notes—literally one ‘against’ another—to construct entire works consisting of musical ideas juxtaposed in tasteful balance. Theorists roughly contemporary with Byrd do not offer us terminology that we would like them to help us precisely and insightfully describe his compositional process. Modern writers do not fare much better, using loosely-defined taxonomies and terminologies. Here, it is useful to make a distinction between the two: a taxonomy attempts to classify musical entities while a terminology ought to name those classes in a consistent manner. While one taxonomy might be useful, its concomitant terminology might be idiosyncratic, and vice versa. Rationalising Byrd’s counterpoint entails identifying the advantages and disadvantages of current taxonomic and terminological schema, and reconciling the various historical and modern schema which have been used to date. A new quantitative model of his contrapuntal technique ought to be designed with an understanding of the appropriate
analytical categories, in order that the data it generates be ultimately ‘useful’ from the traditional perspective. The data ought to suggest something to us, the analyst, as to how (and perhaps why) his counterpoint is constructed in this particular way rather than any other.

1.1.1 ‘Point’ and fuge

The theorist closest in time and tradition to William Byrd (c.1540–1623) is Thomas Morley (1557/8–1602). Morley is believed to have studied with Byrd in the mid-1570s. Morley was so deeply impressed by Byrd’s musical acuity that his dialogic musical treatise *A Plaine and Easie Introduction to Practicall Musick* (1597) bears its dedication to the ‘most excellent Musician Maister William Birde.’ With this evidence of the deep indebtedness of Morley to Byrd, the contents of his *Introduction* are held by some to be the most accurate available record of Byrd’s teachings on music, albeit at second hand. The *Introduction* has much to say on the topic of counterpoint, its second book of three dealing with this topic exclusively with an abundance of pedagogic examples. In particular, Morley sets out to instruct the apprentice in the ways of making one’s ‘point’ and ‘keeping it well’. In ‘point’ we find a useful vernacular term for the formal sub-units of polyphonic works by Byrd and his rough contemporaries.

The first mention of the word ‘point’ in the *Introduction* in the polyphonic context comes as Morley’s figurative Master gives his socratic interlocutor Philomathes a peek ahead at the application of the rules of counterpoint to ‘a Point or (fuge).’

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‘fuge’ are treated equivalently in several other parts of the text. As to what fuge stands for, Philomathes is told that ‘we call that a Fuge, when one part beginneth and the other singeth the same, for some number of notes (which the first did sing)’: a formal device, consequential entries of similar material in counterpoint. Yet, ‘point’ is used elsewhere to describe the motivic matter that the fuge is based on. Morley’s use of same word, ‘point’, to describe both the part (the motivic matter) and the whole (the fuge) is the intentional deployment of synecdoche. Morley’s figure of speech seems to endorse an analytical approach that would seek to demonstrate relationships between the polyphonic edifice and the motivic matter of which it is built. To summarise what we may take from Morley’s Introduction: fuga (simply fuge in another orthography—we may be as fluid as Morley and his contemporaries in this regard) will be used to describe the compositional process, and ‘point’ the output of that process and/or the musical material upon which it is based.

1.1.2 Historical and modern concepts of ‘imitation’

Turning to modern critical voices, in The vocal polyphony of William Byrd (1949) H.K. Andrews offered a threefold categorisation of Byrd’s points: single subject imitation, double subject imitation and rhythmic imitation. However, we know that Byrd not only adjusts the melodic intervals of his points, but also independently augments and diminishes their rhythmic note values, ad libitum. Rhythm is never a necessary invariant in Byrd’s fuga. The consort music, as will be seen, is replete with such rhythmic variation. Andrews’s categories are not successful in part because they are not mutually exclusive

5. Ibid., 96, 130.
6. Ibid., 76.
7. For example, reference to an exercise printed in which the tenor ‘expresseth the point, the base reverteth it.’ Ibid., 162.
(there is no clear division between Byrd’s ‘single’ and ‘double’ points), and in part because they do not reflect the rhythmic diversity of Byrd’s points. The terminological confusion may have begun when the word *fuge* was consistently changed to ‘imitation’ in R. Alec Harman’s 1952 edition of Morley’s *Introduction*\(^8\). Yet Morley uses the word ‘imitation’ elsewhere in the original to refer to the copying of a musical style, not to *fuga* at all.\(^9\) No use of the word ‘imitation’ in the original could even charitably be interpreted as referring to a musical process on the same level as *fuga*.

What may have motivated Harman’s rewording is a knowledge of Zarlino’s fourfold (and not mutually exclusive) categorisation of contrapuntal process into *fuga* (*legata* and *sciolta*) and *imitatione* (likewise, *legata* and *sciolta*). Zarlino’s definition of *imitatione* in his counterpoint treatise seems to permit the kind of intervallic flexibility permitted in Morley’s comments that ‘in Fuges wee are not so straitly bound’ compared to what we today call strict canon.\(^10\) Morley’s *dictum* is central to the understanding of the mindset with which a composer of Byrd’s training and inclination would approach *fuga*: entirely free to modify the intervals in his entries as he saw fit. Morley’s permission here meant that the composer did not have to abide by the formal parameters inherent to a given subject, parameters such as entry distance and transposition level. On the contrary, the composer could bend and deform the subject as he saw fit, to generate points of significantly greater interest than might deemed viable using strict deployment of the subject. This is notion is at the heart of Morley’s conception of *fuga*.

It is not rigorous to privilege the Anglicisation of a term from Zarlino over the perfectly useful terminology of Morley’s *Introduction*, simply because of the attractive-

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9. See, for example, Morley, 312.
ness of enforcing an artificial consistency of terminology across languages. It is not sensible to introduce a dependency on Zarlino’s taxonomic preferences into the discussion of Byrd’s polyphony, lest they be later shown to be an incomplete understanding of contemporary polyphonic practice. Zarlino’s approach to the topic is hardly above reproach, not least in his confused categorisation of polyphonic types. Furthermore, Zarlino’s ideal composer was Adrian Willaert (1490–1562), a composer whose straightened and periodic fugal constructions are quite far removed in terms of flexibility from those of Byrd. Quite apart from all those issues, Morley (and thus likely Byrd) just did not once use the word ‘imitation’ in the sense that Joseph Kerman, Oliver Neighbour and Richard Rastall have done. Harman’s grip is apparent in the writings of all three, from time to time, though it is unlikely that they did not know of the translational discrepancy. Morley’s use of ‘imitation’ is strictly limited to referencing the hackish plagiarism of musical material or style, typically in a pejorative sense. Whatever connotations it has today of strict replication and being ‘straitly bound’ are not appropriate to describe the process in which Morley educates his reader. ‘Imitation’ should no longer be considered an appropriate word, for both historical reasons and for the confusing connotations it seems to have wrought upon writings on Byrd’s counterpoint since the enterprise began in earnest.

1.1.3 Towards a new vocabulary

The title of Davitt Moroney’s unpublished paper, ‘What’s the point? Non-imitative imitation in Byrd’ neatly summarises the absurdity of the state of affairs up until recently.11 Using the term ‘imitation’ to describe the construction of polyphonic points

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in which, sometimes, no two entries of the subject are identical is simply inappropriate. At that conference, Morley’s word *fuge* was advocated during a panel session on contrapuntal technique and rendered (perhaps confusingly) as *fuga*, and recommended by Milsom as a valuable addition to the vocabulary of Byrd analysis. Moroney’s observations in 2005 contributed to a general trend in the discourse towards new terminology. For instance, he adopted the evocative term ‘morphing’ to describe the flexibility of Byrd’s *fuga* subjects. There were, however, previous efforts to arrive at suitable terminology for the kind of freedom which Moroney is driving at. In a pair of analytical papers Joseph Kerman introduced the polyvalent term ‘cell technique’, which describes successive motives treated shorter than the typical subject, treated in *fuga* without the strict preservation of interval content, which then coalesce into a definitive statement of the subject. Later, Kerman revised his working definition of ‘cell technique’ so that it may be either ‘homophonic, half-homophonic, fully imitative or even strictly canonic.’ James MacKay comments that Kerman’s later conception of the ‘cell’ is loosely-defined ‘so as to capture the potential flexibility of Byrd’s contrapuntal procedure.’ Looking at it less charitably, Kerman’s definition is so permissive that it can stand for any kind of development of small, repetitive musical units that the analyst sees fit to focus on. It is not a particularly illuminating addition to the terminology applicable to Byrd’s points though it is certainly part of the effort to define appropriate terminology for that purpose.

Milsom’s recent study of 16th-century continental polyphony provides a most useful, though idiosyncratic, lexicon for describing some common constructs in polyphony of this era.\(^{16}\) Milsom introduced and embraced the term *fuga*—hinted at above. In so doing, he reclaimed the flexible contrapuntal behaviour of the time, not as an intermittent diversion from a controlling rubric of strict imitation, but as the controlling strategy itself, an observation equally applicable to Byrd’s contrapuntal technique. Milsom has also contributed the term ‘stretching’ to the lexicon of *fuga* analysis: the independent rhythmic augmentation and diminution of individual notes of a *fuga* subject. In a later article, Milsom parses the polyphonic surface of Lassus’ motets into simple contrapuntal combinations termed *fuga* cells which appear in various viable ‘interlocks’ (or superimpositions).\(^{17}\) Julian Grimshaw has recently applied Milsom’s lexicon to English repertory, and in a most recent article sketches the use of subject shapes known to older composers in the early motets of Byrd.\(^{18}\) Another useful one of Milsom’s terms is aired in this dissertation and applied to analyses of Byrd’s motets: ‘flexing’ or ‘how the interval content of a subject might vary from voice to voice’.\(^{19}\) Accounting for Byrd’s flexing and stretching in a quantitative way is at the very core of this dissertation. Milsom’s terms are used throughout and definitions of perhaps unfamiliar terms defined here for the first time, or introduced in the studies of Milsom and Grimshaw may be be found in Appendix A.

1.2 On the fantasias

Before a formal model for measuring what will be called ‘subject deformation’ is offered, we must consider previous writings on the consort music and the prospects that they suggest for a new approach in conjunction with this new terminology. Writings on the instrumental consort music of Byrd are limited in quantity and tend not to broach substantive analytical questions in the manner that characterises exemplary writing on Byrd’s vocal polyphony. From the two principal authors, Oliver Neighbour and Richard Rastall, we gain some analytical leads into the overall shape of Byrd’s consort music, but not the local contrapuntal strategy. Byrd’s instrumental music can be broadly categorised into that for keyboard instrument and for (viol) consort. Of the consort music, a number of sub-categories roughly apply:

1. Fantasias
2. Ground-bass compositions
3. Dances (Pavans, Galliards)
4. Cantus firmus compositions (In nomines and various hymn-tune settings)

Without a ground bass, a cantus firmus or duty to a text, most of the fantasias are written in free fugā. Those (longer) fantasias that do not consist exclusively of fugā are Fantasia a 6/g1 (BE 17/12) and Fantasia a 6/g2 (BE 17/13), to each of which is appended a galliard. When writing in the genre of the fantasia (‘Fantasy’) as Morley understood it, the composer could ‘take a point at his pleasure and wresteth and turneth as his list’ and ‘is tied to nothing, but that he may add, diminish and alter at his pleasure.’

It can be supposed that Morley’s attitude to the fantasia was derived from Byrd’s, and that these works for consort offer an insight into exactly those kinds and manners of additions, diminutions (stretching) and alterations (flexing) that pleased their composer so.

In his volume on the keyboard and consort music, Neighbour firmly contextualised Byrd’s consort music as a stylistic consequent of the consort music of Tye, Sheppard and White, amongst others. Indeed, he mentioned few works of Byrd without reference to older composers. The contrast between the straightened polyphony and conservative form of Byrd’s precursors and the relative freedom of Byrd’s writing is heightened by these juxtapositions. Descriptions of form tend to shape Neighbour’s discussions of the works. The analysis by ‘phase’ of 6g/1 is to be contrasted with the more disparaging use of the term ‘paragraphing’ to describe the Fantasia a 6/f (BE 17/11).21 From language elsewhere in his analyses it is clear that Neighbour viewed 6/f as the least successful of the works for larger consort. Neighbour correctly identifies that in untexted works, the composer must rely solely on cadential action to delineate sections within a fantasia, but focuses on the larger picture to the neglect of the local contrapuntal processes that make up each ‘section’ as he identifies them.

When Neighbour does tackle the content of each point, he repeatedly finds cause to draw sharp distinctions between passages that are ‘broadly harmonic in effect’ and those that are ‘imitative.’22 These distinctions are not always borne out by the musical evidence. The eleventh variation, for example, of Browning a 5 (BE 17/10) is in fact no less of an example of fuga than the ten which precede it, contrary to Neighbour’s

22. Ibid., 71.
assertion. Another misstep arises in the discussion of Fantasia a 4/g (BE 17/4), in which Neighbour refers to homophonic ‘interludes’ between polyphonic points which turn out in fact to be points of fuga themselves. This is explicitly redressed in the model analysis of this piece below.

Neighbour identifies the fluid approach to counterpoint that Byrd exhibits in the consort music, writing ‘he entertained no fixed notions of the forms that imitation should take.’ He goes on:

the result is a technique that is no longer imitative in the accepted sense, and may perhaps be called ‘figural’. Short figures in which the constants are the rhythm and the general shape rather than an exact sequence of intervals pass freely from part to part.

Yet even Byrd’s rhythm is not necessarily as fixed as Neighbour’s comments would imply: herein an echo of H. K. Andrews’s faulty category of ‘rhythmic imitation’. This passage is Neighbour’s principle acknowledgement of Byrd’s flexible fuga. In the paragraph immediately following, Neighbour goes on to speak of ‘the first section’ of Byrd’s Browning as ‘the province of imitation proper.’ Such a dichotomy between a normative “imitation proper” and ‘figural’ or flexible counterpoint is to be questioned. Challenging the prevalence of the term ‘imitation’ and adopting fuga, as mentioned above, reclaims flexibility as the norm, not as occasional deviant contrapuntal behaviour. Neighbour’s volume includes the only exhaustive study of the consort music, however, and each model analysis below takes the most valuable insights of Neighbour as its starting-point.

23. Ibid.
24. Ibid., 70.
25. Ibid., 71.
26. Ibid.
Richard Rastall is has treated at least two fantasias to extended examination since Neighbour’s book. His first study considers the Fantasia a 6/g1 (BE 17/12). In his first study, Rastall’s declared intent is to recover the status of 6/g1 as a work deserving of consideration commensurate with its thematic and modal companion, the Fantasia 6/g2 (BE 17/13) which Byrdelected to publish in Psalms, songes and sonnets (1661). Rastall takes Neighbour’s capsule analysis as a base, but provides his own dissection of the fantasia.\(^\text{27}\) Rastall frames his discussion with a very worthwhile question (inter \textit{alia}): ‘What is the work’s basic structural principle, and how successfully did Byrd use it?’\(^\text{28}\) Rastall argues that each section of 6/g1 exhibits a different controlling technique, one of imitation, antiphony, quotation (Greensleeves) or, most curiously ‘phrase-repetition.’\(^\text{29}\) Rastall has also provided a more recent analysis of his own reconstruction of the Fantasia a 4/G, which exists only a single part in consort form (published as a fragment in the Appendix to BE 17).\(^\text{30}\) Byrd set the text ‘In manus tua’ to the fantasia for publication in his Gradualia of 1607, and Rastall has reconstructed the consort setting from this source. The analysis of 4/G succeeds in demonstrating the motivic connections between the points used by Byrd to construct the fantasia, suggesting an line of inquiry into similar connections in other fantasias. Both analyses, depend heavily on harmonic readings of the texture, and to this end Rastall provides a so-called ‘fundamental bass’ part as adjunct to his examples to demonstrate ‘moves decisively from tonic to dominant’ in 4/G.\(^\text{31}\) Liberal use of scare quotes (“modulation”) insures

\(^\text{27}\). Incidentally, Rastall disagrees with Neighbour’s chronology of the works. This is not of immediate importance to us.


\(^\text{29}\). Rastall, ‘William Byrd’s String Fantasia 6/g1’, 153. This last category is a coinage referring to the passage of a melodic figure two voices with little or no variation.


\(^\text{31}\). \textit{Ibid.}, 15.
Rastall somewhat from being accused of an anachronistic tonal reading of the fantasias. However, unpicking the role of ‘strong’ cadential progressions in demarcating intermediary tonal centres is important to understanding these works as more than just ‘a serial collection of tunes,’ as Rastall correctly observes.\textsuperscript{32}

1.3 Prospects for the analysis of the consort music

Stretching and flexing of \textit{fuga} subjects are but two types of compositional operations which are considered here as specific types of class of musical operations called ‘subject deformations’, a geometric analogy which seems apt if pitch-space and duration are considered as the two principal orthogonal axes of musical space. By adopting this metaphor we are invited to consider the \textit{fuga} subject as a musical entity always in a state of flux, in which the exact rhythmic and intervallic profile is not always the key to its persona. Rather, as a coffee cup is identical to a doughnut in the topological sense (both are toroids), so are two deformed entries of the same \textit{fuga} subject—despite their superficial dissimilarities. The entry maybe be twisted and distorted, contracted and dilated, but its identity—for the purposes of \textit{fuga}—stays roughly the same. The analyst who doggedly searches for instances of ‘imitation’ understood as exact intervallic and rhythmic replication in Byrd’s \textit{fuga} will be disappointed. The idea that Byrd’s freedom in \textit{fuga} is just either deviant or simply a secondary consequence of other compositional concerns ought to be set aside. Variety in \textit{fuga} entries is central to the composition of his points. Grimshaw notes that in the larger motets which he studies, this variety is sometimes a simple consequence of the vertical exigencies of writing for five and six parts: doublings are the inevitable destination of individual part-writing in such large

\textsuperscript{32} Ibid., 14.
textures. In works for fewer voices, however, he sees the potential for true ‘developmental fuga.’ The three- and four- part fantasias are likely harbouring evidence of this relatively recent take on Byrd’s contrapuntal strategy. By abandoning the misapplication of the analytic category of imitation and by adopting the taxonomies of the most recent authors who hold Byrd’s very variety is his norm, we are well-grounded analytically for the next step in the process: designing a measure of subject deformation in order to reify analytic prose propositions as quantitative hypotheses.

33. Grimshaw, ‘Fuga in Early Byrd’, 259
Chapter 2

A quantitative method for the study of fuga

2.1 Previous work

Nigel Nettheim’s bibliography of statistical applications in musicology is an exemplary survey of mathematical and statistical applications in music (up to 1997) by a statistician-cum-musicologist who does not shrink from identifying ‘several cases of misapplication of statistics.’\(^{34}\) Beran’s 2004 monograph *Statistics in Musicology* is an unrelentingly formal account of (sometimes advanced) statistical techniques that have had fruitful application in interdisciplinary studies with a music-analytic bent.\(^{35}\) The possibilities of statistical applications to music are broached in some contributions to Clarke and Cook’s *Empirical Musicology: Aims, Methods, Prospects*, but the tools described have aged badly since its publication in 2004.\(^{36}\) A number of academic associations periodically publish articles connected with empirical studies based on transcribed symbolic representations of scores, though often not exclusively.\(^{37}\) As the principal development offered here is a new technique for measuring the diversity of successive en-


\(^{37}\) *Empirical Musicology Review, Journal of Mathematics and Music*
tries of *fuga* subjects, work in the field of symbolic melodic similarity is of interest. The convenors of the annual Music Information Retrieval Evaluation eXchange (MIREX) annually invite submissions of algorithms designed to identify similar melodies to a base set of queries on the Essen song collection (the Symbolic Musical Similarity task). The most recently top-ranked submissions adopt a shape-based approach to comparing melodies.\(^38\) A relevant technical paper published in a 2011 volume of the Springer-Verlag series ‘Studies in Computational Intelligence’ describes a number of melodic dissimilarity measures and technical adjustments to them which improve their utility in practice.\(^39\) In particular, this paper describes some of the different types of melodic dissimilarity measures in formal terms, which provides useful grounding for the design of a new measure.

The application of statistical methods to the music of William Byrd is not without precedent. In 1985, Hilde Binford used the statistical chi-square test to assess the significance of stylistic differences between anthems by William Byrd and Orlando Gibbons.\(^40\) Binford performed a census of chord qualities, the use of ‘first inversions’, bass-line motion of ‘root position’ harmonies and of melodic intervals of the tenor and soprano lines. This data is promptly pressed into service to confirm the author’s preconceptions that ‘listening to a varied collection of Gibbons’ works, it is easily noted that Gibbons is experimental with the melodies.’ Her analysis, conveniently, ‘confirms this empirical observation.’\(^41\) As an early application of statistics for stylistic comparison involving a model of Byrd’s melodic and harmonic preferences Binford’s thesis is

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notable.

In John Morehen’s 1992 study, nine *opera dubia* are compared against a feature set extracted from the significantly larger body of Byrd’s printed Latin music. Morehen has extracted ‘11,328 musical features’ from the 75 authenticated motets in the control group, though only ‘1,026 [of these features] were found to be present in either the dubious pieces or in the model.’ The implications of the massive redundancy of the model of the authenticated works are not explained. Furthermore, Morehen offers Pearson correlation scores for each of the eleven works under consideration, and, convincingly, the features of the works by Parsons and Victoria also tested are not strongly correlated to the features of the authenticated Byrd works. Pearson scores measure the extent of linear correlation between two variables. Morehen does not explain if that assumption of the model is satisfactory, although it seems to be the most useful, *post hoc*. Methodological opacity in Morehen’s work undermines a study which has otherwise thoughtful insights into the challenges of modeling Renaissance polyphonic style in particular.

David Wulstan has interpreted the results of computational methods applied to Byrd’s music by the late John Duffill. The appendices to Wulstan’s chapter ‘Byrd, Tallis and Ferrabosco’ draw on Duffill’s analyses of the pitch frequencies and voice ranges across the known church polyphony of Byrd. Like Binford, Duffill used the chi-square statistic to evaluate the significance of differences. Some comparisons with data from works by Tallis and Ferrabosco are used to support the less statistical argument

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in the main body of Wulstan’s text. Though the data dates from as far back as 1991
the results of Duffill’s and Morehen’s studies are the most recent analytical attacks on
Byrd’s music that are primarily statistical in nature. Since then, significant advance-
ments have been made in the tools available to musicologists for processing symbolic
representation of musical scores, and it is time to revisit Byrd and examine the ways in
which the use of these new tools can enlighten our understanding of his polyphonic mu-
sic. None of these studies directly tackles the issue of flexibility of Byrd’s subjects and
are more oriented towards large-scale corpus studies than the present study. It is hoped
that returning to the positivistic principles which informed the short burst of studies in
the late 1980s and early 1990s in the light shed by the most recent work on Byrd’s fuga
will show this line of enquiry into the composer’s output to be fruitful once again.

2.2  Designing a deformation metric for fuga subjects

2.2.1  Motivation

Quantifying the way Byrd varies (deforms) fuga subjects in in his points allows us to
describe with statistical evidence their otherwise indescribably volatile nature. This
section describes attempts to construct a model for such calculations and comparisons.
This data is then available to the analyst to support prose hypotheses. A contrived ex-
ample suffices: ‘the final points of Byrd’s fantasias are relatively stable.’ Once we have
a suitable measure, we simply take a cross-section of Byrd’s fantasias (all, if possible),
and statistically assess the average amount of deformation in their final points, and its
significance. To define the measure, we take formal definitions of the compositional
operations that have been observed to act on fuga subjects, after Milsom, and define a
new measurement of ‘subject deformation’ which accounts for observed instances of
those operations. Such a casual definition invites the question: deformation with respect to what, exactly? Byrd’s points are frequently peppered with so many different forms of the subject that it is impossible to identify an unmodified entry of the controlling subject. One approach is to take the first entry as the definitive version (hereafter ‘plain form’) of the subject. To do so seems unduly arbitrary, since it presupposes that the process of fuga works away from a starting point. Perhaps fuga is more about the destination, the final entry? Perhaps it can be about both the beginning and the end? Nominating the first entry as the unmodified form is also problematic if it is very short, compared to the rest of the entries—especially if the other entries share a distinctive motive. Why not then dispense altogether with the supposition that such a plain-form of the subject exists at all? ‘Subject deformation’ is thus defined as the amount by which any one entry of a subject subject is different from any other, according to a restricted set of possible variations. This definition reflects the diversity of subject deformations and does not require the existence of an unmodified form of the subject to make sense. Indeed this definition does not preclude the possibility of heuristically determining candidates for an ‘unmodified’ form at a later stage.

2.2.2 Implementation

We define ‘subject deformation metric’ as a measurement of the effect of the transformations referred to as flexing and stretching. Flexing, as Milsom defines it, is the modification of the melodic intervals of a subject, while stretching is the augmentation or diminution of one or more note values in length independently of the others. Since the simple metric defined here aims to account only for these two kinds of deformation, we can strip down the symbolic representation of the subject to a pair of numeric values for each note, its pitch and its duration. To determine if the first subject can be flexed
into the second, we iterate through each melodic interval of both subjects and compare them. For every different interval (whether greater or smaller) we increment the value of the metric by one. To determine if the first subject can be stretched into the second, we iterate through each note of both subjects and compare their durations. If these two durations are not equal, we increment the metric by one. We notate this measurement as $D(e_i, e_j)$, where $e_i$ and $e_j$ represent any two entries, indexed by an integer which denotes the position of the entry in the fugue. Thus, the first entry in a point is denoted $e_1$, the next is $e_2$ and so on. The subject deformation metric between the first and the third is denoted $D(e_1, e_3)$, for example.\textsuperscript{44} This procedure can be repeated between all pairwise combinations of entries that appear in a contrapuntal point.

While there are many statistics of interest that can be extracted from this data, we focus on two. First, summing the total deformation metric between one subject and all others we get an indication of which subjects are more deformed than others. The entry or entries with the lowest total deformation metric—which gives the form that is the “least far from most” on average—will be called the ‘plain-form candidate(s).’ Second, we study the deformation metric between successive pairs of entries. This gives an indication of how the rate of deformation increases or decreases throughout the point. This collection of metrics will be called the ‘successive deformation profile’ of the point. These two statistics taken together can then be used to generate graphical representations of entries, where the deformation metric between successive subjects is indicated by the number of lines connecting each successive entry. These graphs, in turn, motivate analytic explorations of the musical text. These data will reappear in the model analyses.

\textsuperscript{44} The use of $i$ and $j$ is a mathematical convention which stands for any valid index integer'. It allows us to reference the function that returns the subject deformation metric in a general way, without mentioning any of its arguments explicitly.
2.2.3 Limitations and adjustments

Note that this metric does not make sense when entries of different lengths are considered.45 Therefore, the metric as implemented only considers the first notes of both subjects up to a specified cut-off point. This is called the window of the deformation metric, denoted by \( w \). The notation defined above can be thus adapted to include the window of the metric. Thus, \( D_w(e_i, e_j) \) represents the metric in a general sense, while, concretely, \( D_w(e_5, e_6) \) means the subject deformation metric between entry 5 and entry 6 with window \( w = 5 \). It can be shown that increasing the value of \( w \) after a certain cut-off point does not significantly alter the relative ranking of subjects by this measure, but this depends on the subject in question. Choosing an appropriate value for \( w \) is an important part of the methodology, which is described later.

In order to account for stylistic commonplaces which would have otherwise appeared as intentional deformations in the metric results, the metric has been adjusted in two ways. First, owing to the relative frequency with which the first note of fugue entries are stretched, not only in the repertoire of Byrd but throughout common fugue practice, any stretching of the first note of an entry is discounted by the adjusted metric. Second, we take account of the fact a transposition preserving a written scale step of one degree can result in the reduction of the interval size. For example, an entry starting F–G–A–B\( \flat \) shifted up a major second to start G–A–B\( \flat \)–C has a different set of melodic intervals (the last interval in both is different) and this would have been considered a flex by the original version of the algorithm. Of course, no intentional flex has occurred here, the interval profile is different by virtue of transposing within the pervading mode. Thus, flexes from minor seconds to major seconds and vice versa (and only those) are ignored

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45. Technically speaking, the function is not defined for entries of unequal lengths.
in the adjusted metric.

2.2.4 Worked example

The clearest way to see all these new concepts in action is to provide a worked example of the measurement process from beginning to end. The point under consideration is the second point of *Fantasia a 3/c1* (BE 17/1). The window size \( w = 5 \) for the purposes of this demonstration. This is the largest window size that allows us to make successful measurements between all entries. (The shortest entry is six notes long).

The script calculates the deformation metric with \( w = 5 \) between every single pair of entries in the point. The entries had been previously extracted from the score by hand and input into a MusicXML file with eight staves (one for each entry). The results of each pairwise measurement are summarised in Table 1. To find the deformation metric between any two subjects, we follow the table exactly as one consults a cross-tab, finding the intersection point between the row and column containing the subjects we want to get the deformation metric for. For example, \( D_5(e_3, e_4) = 3 \). Note that \( D_5(e_i, e_j) = D_5(e_j, e_i) \) for any choice of \( i, j \). This property of the subject deformation metric is called symmetry. The final column of the table shows the row-wise sum of the deformation metrics for each subject, and all of the other subjects, exactly once. Since the metric is symmetric, the column-wise sums are identical to the row-wise sums, hence the omission of the column-wise sums in Table 1. The subject with the lowest values in this column are the plain-form candidates, being those subjects which are the least deformed, on average. Another way of thinking about this is that the plain-form candidates require the least amount of deformation on average to be turned into a randomly selected subject in the point. The circled figures indicate the readings that make up the successive deformation profile for this point: \( \langle 0, 0, 2, 2, 0, 3, 0 \rangle \). This gives
us the deformation metric between $e_1$ and $e_2$, $e_2$ and $e_3$ and so on. In other words, it tells us how many subject deformations are required to transform the first entry into the second, the second into the third, and so on.\textsuperscript{46} These two new concepts (plain-form candidate and successive deformation profile) recur throughout the analyses, though the deformation table from which they have been determined will not be provided in every case.

2.3 Prospects for a quantitative approach to Byrd’s \textit{fuga}

Julian Grimshaw sketches the shift in focus which characterises the efforts of the last half-century in understanding Byrd’s contrapuntal strategy.

[Kerman’s study ‘Byrd, Tallis and the Art of Imitation’] made a significant step forward because it went further than was typical in the standard textbooks—concerned primarily with broad issues, such as classifying fuga as strict or free, with real or tonal answers, and so on—to a consideration of the internal logic of fuga, with more emphasis on how the voices within an exposition relate to one another.\textsuperscript{47}

A new quantitative approach, then, ought to illuminate the ‘internal logic’ of Byrd’s points. As the point progresses, how are logical relationships between the part and whole expressed? Does the harmonic surface reflect these relationships? Using a symbolic representation of the score, we can computationally track the dilation and contraction of any measurement of the polyphonic structure that we care to construct. Perhaps Byrd’s cadence points are accompanied in the main by increased ‘flexing’ and ‘stretching’? After all, Morley wrote that ‘if your descant should be stirring in any place, it

\textsuperscript{46} Due to symmetry, again, we may as well have chosen the diagonal directly above the zeros which bisect the table, which would return the same series of integers.

\textsuperscript{47} Grimshaw, ‘\textit{Fuga} in Early Byrd’, 251.
should bee in the note before the close.'\textsuperscript{48} Or perhaps this refers to a climax-complex of register, \emph{stretto} and subject deformations all working together in synchrony to shape each point, as Kerman demonstrates in ‘Tribue, Domine’ from the \textit{Cantiones Sacrae (1575)}\textsuperscript{49}

Answering these questions once and for all is not within the purview of this study. Finding and defining their analogs on the quantitative plane, however, is. A handful of new measures have been proposed here, and are applied to a small selection of instrumental works by Byrd to investigate their applicability. In particular, a simple ‘subject deformation metric’ has been proposed. This can be used to determine heuristically the least-deformed statements of the subject by calculating the plain-form candidates, and to track deformation rates as Byrd’s points play out using the subject deformation profile. The fantasias are the first test-bed for this approach. It would be rash to claim to have discovered evidence of compositional strategy that characterises a composer from the results of tests carried out on a small subset of works. However, the methods offered here are theoretically scalable, much like the results of any investigative laboratory experiment. The proposed measures could serve as the essential tools in a much longer longitudinal corpus study, in stylistic comparisons with the music of Byrd’s contemporaries or in studies of authenticity. Of course, Byrd is not the only composer whose style is germane to these methods. Much as Milsom’s exposition of his work in the field dealt with continental sacred polyphony of Lassus, Clemens non Papa and Crecquillon, the techniques described here are not exclusively applicable to William Byrd. For the moment, however, Byrd’s fantasias move into focus as they yield to the approach described in the two chapters above.

\textsuperscript{48} Morley, 81.
\textsuperscript{49} Kerman, ‘Old and New in Byrd’s \textit{Cantiones Sacrae}’
Chapter 3

Model analyses of the consort music

3.1 Analysis of *Fantasia a 3/C1* (BE 17/1)

The longest of the three extant three-part fantasias (at 46 modern bars in length), the work proceeds in three imitative points. The first is the briefest of the three (bars 1–10). The second is marginally longer (bars 10–23) and closes on G major. The third and final point is as long as the two which precede it combined (bars 23–46). A more active subject with prominent crotchet movement and fifteen entries seems to address the potential tedium of a point that is dwelt on twice as long as those which precede it. The fantasia concludes on C major. Apart from a brief observation concerning the rising and falling fourths which unify the thematic material of this fantasia, Neighbour curtails his discussion of this fantasia, with the finding that is a ‘beautiful piece of unhurried eloquence.’

3.1.1 First point (bb. 1–10)

This short point demonstrates very little subject deformation. The only two entries to be deformed are the second entry (in augmentation) in III bar 1, and the final entry in

50. Neighbour, 99.
II bar 8. In the first case we can imagine the stretching backwards of the first semibreve of the second entry to a breve, and speculate that Byrd’s initial sketch for this point involved noting the valid ‘self-interlock’ at the lower octave between the subject and its augmentation. As for the last entry, it turns out that Byrd has retained the interval structure of the subject as the entry distance decreases by deftly stretching the subject, such that it resembles the entry in augmentation. A clever ornamentation keeps the descending fourth of the subject in play as the subject’s descent to E supports the very weak close.

3.1.2 Second point (bb. 10–23)

Because this point has relatively few entries for its length, it provides clear examples of some of the functions that voices which are not presenting the subject perform in Byrd’s fuga. ‘Shadowing’ has been defined by Milsom to describe the textural thickening of fuga by the inclusion of doublings at appropriate imperfect consonances. Shadows pervade this point at the very first entry (I and III, compound minor third), through bars 17–18 (I and III, compound minor third) and in bar 20 (II and III, minor third). The two notes before the shadowing line in III bar 20 seem to be engineered to give the sense of an entry preempting the final entry on G. Highlighting non-subject, non-shadow lines is revealing, as in Figure 1. Non-subject, non-shadow lines are called ‘continuation lines’ by Milsom, and the figure demonstrates the functions that they can perform in the fuga texture. I and III bar 13 is free three-part counterpoint against the subject entry from II bar 12. The destination of III is the cantizans component of an evaded close on A (a variant reading has G sharp in III bar 14). Other cantizans components are indicated with an asterisk. It is clear that continuation lines are not simply undirected and anodyne free counterpoint, but rather serve to intensify the cadential figure in this case implied
by the C-B descent of the subject, even in inner voices. The characteristic rising fourth of the plain-form candidates is flexed (shown with an angle bracket) to a minor sixth in II bar 16 specifically to facilitate hints at a cadence on G. The continuation line in I bars 20–22 is an unsophisticated circumnavigation of G, sitting on this pitch until the last two entries of the subject confirm that the point closes on G.

3.1.3 Third point (bb. 24–end)

Understanding this point using the deformation metric shows the importance of establishing an appropriate window size in order to retrieve truly useful information about the point. As explained above, the subject deformation metric requires a window parameter which determines the portion of the subject (from the start) which we use to compare the two subjects. Of the fourteen entries with \( w = 4 \), there are eleven plain-form candidates (three exceptions), but setting \( w = 8 \), there are only nine (five exceptions). Expanding the window size in this way demonstrates that our view of the point changes as we compare larger windows of the subjects. So which is the more accurate view? Looking at all the entries together in Figure 2, few would agree that examining the first four intervals (five notes) could tell the whole story of the point. The model surely must consider the conspicuously common rising scale in crotchets, and what happens after it. Thus it seems that \( w = 4 \) is out of the question.

The choice is discretionary, not arbitrary. Choosing a window size does have an impact on the data that is returned by the metric, and has a consequential effect on our analytical conclusions. The question ‘how much of an entry is relevant to the subject?’ does not yet have a programmatic solution, but in the interim we can be guided by our musical sense. For example, the step-wise descent of the tail of the second entry is stylistically more prosaic than the downward leap after the scalar ascent that seems to
characterise a large number of the entries. $w = 9$ has been chosen for the pragmatic reason that it allows us to include the shortest subject in our analysis without the deformation metric becoming undefined (see 2.1 Designing a deformation metric) and for the additional musical reason that such a window size includes the relevant downward skip which is mentioned earlier. With $w = 9$ there are eight plain-form candidates. It is notable that plain-form candidates appear at the opening and the close of the point, suggesting that deformation is a process that operates during the middle phases of a point. Additionally in this point, continuation lines which resemble an entry in their shape are noticeable in III bar 30, I bar 36 and III bar 39. It is as if Byrd uses as much of the subject as is possible (sometimes this is only two or three notes) to explore interlocks between entries which would not otherwise be valid without substantial reworking of the identity of the point. The point can be summarised graphically using an entry diagram that has been enriched with the successive deformation profile data, shown in Figure 3. Each circled letter represents an entry and the starting pitch of that entry, while the lines connecting each circle indicate the subject deformation metric between each pair of entries. A dotted line represents a value of zero (or no deformation required), between one and four lines represents those values respectively and the thicker and thickest lines, values of six and seven respectively. Hence, the graph shows the following successive deformation profile: $\langle 0, 0, 0, 2, 2, 3, 4, 1, 3, 0, 7, 6, 0 \rangle$.

3.2 Analysis of Fantasia a 3C/2 (BE 17/2)

This fantasia consists of five points of free fuga, the end of the first point very clearly overlaps with the start of the second, which has the effect of undermining any strong cadence on C. The discursive second point—having visited A—ends unambiguously on
an E major root position sonority, the most significant structural subdivision so far. The
fourth point snaps back immediately to the unmodified major mode on C, but moves
rapidly towards a very prominent A major sonority in bar 21. The final point, the
longest, begins with simple four-note subject which expands out a into six note sub-
ject after treatment in fuga to bring the fantasia to close on C major.

3.2.1 First point (bb. 1–5)

The first short point of this fantasia is very concise, with only two forms of the sub-
ject in play: a form starting on G in I and a form starting on C in III, one breve later.
The principal difference between the two forms is the adjustment is clever rhythmic
stretching to suitably align the final destination of the second entry (E–D–C) with the
the start of the third entry on G in I bar 2. Due to the regularity of entries and because
of the foursquare length of the subject (four breves) notably little use is made of con-
tinuation lines to pad space between entries. The plain-form candidates for this point
are entries 1, 2 and 3—all at the start of the point. The entry distance expands as the
point concludes, contrary to the more common contraction which defines stretto. The
decorative figure in III bar 5 implies a close on C, but by this stage a distinctly different
fuga subject has entered in I on G. Such overlapping is not uncommon in Byrd’s points.

3.2.2 Second point (bb. 5–12)

From start to finish, the material of this point can be related to the controlling shape of
a small downward move, a leap up, and two–three step-wise descents. Figure 4 shows
this point in full and instances of the subject are shown with a brace. The point opens
with the subject once in each voice, cycling through I, II and III starting on G, C and G
respectively as typical in the first phase of the points in the consort music. The rising
perfect fourth of the entry starting on G is flexed to a rising perfect fifth in the form starting on C. Modifying entries related by a perfect fifth (as C is to G) in this way is the typical construction of a tonal answer in the common practice fugue. So doing weakens the implication of the relatively distant tonal area that a perfect fifth above the dominant scale-degree carries. The pitch names of the intervals which are flexed here are instructive. Byrd is not flexing fifths built on G (which would tend to neuter implications of a centre of D and keep us in a C–G complex), but built on E, neutering implications of B and keeping us firmly in an A–E complex. The transposition levels of the next two entries (A and E respectively) support this change of direction and in bar 9, I and II participate in a weak but unambiguous cadential gesture on A. The successive deformation profile \((w = 5)\) of this point is \(\langle 1, 1, 5, 2, 2, 2 \rangle\). We notice the 5 which draws us to note the extensive stretching that facilitates the decoration by appoggiatura of the cadence on A. During the rhythmically more vital second part of the point, a few design elements are of note: overlapping of entries in I bar 10 in a descending sequence, with intervals flexed to the smallest possible distance; the harmonic support by entries in III which retain the fourths and fifths of the first guises in which we met the subject; the construction of the continuation line in II bars 10–11 as a displaced composite shadow of I and III alternately. As the last entry in III concludes with the lowest note heard up to this point (lowest A on a bass viol), I and II descend to participate on a full close on root position E major as insinuated by the flexes so very far back (or so it seems) in bars 5–7.

3.2.3 Third point (bb. 12–19)

This point starts with two pairs of entries on E and A, and A and E, respectively but without the raised sixth which characterised some of the harmonic direction of the
previous point. The three final entries on C, C and G respectively confirm a pivot back to the tonal neighbourhood delineated by the first point. Six of seven entries are introduced with metronomic regularity, once every breve (except for the first, appearing after a minim). Non-subject material in II tends to shadow the outer voices alternately (see II bar 13 for shadowing of III at the third; II end of bar 14–15 for shadowing of I at the sixth below), while continuation lines in the outer voices tend to move in contrary motion to each other, or to the voice presenting the subject (see I and III bar 15). The last entry overlaps with the first entry of the next point, together yielding a fleeting G major sonority.

3.2.4 Fourth point (bb. 19–22)

The plain-form candidates \((w = 6)\) for this short point are the first two entries on E. The successive deformation profile for the five entries is \(\langle0, 0, 1, 5\rangle\). The first deformation is the chromatic alteration of the third note of an entry starting on A, raising C natural to C sharp in I bar 20. The resultant sonority is an A major triad, and Elliott’s edition precedes that immediately with a G sharp in II bar 20, intensifying the emphasis on A. However, Elliott’s choices here are a conflation of two sources: no source for this fantasia has both alterations at once.51 This is a salutary warning against the use of any measure that is programmatically implemented without first confirming the reliability of the source materials. Accidentals and musica ficta pose significant, but not insurmountable challenges to a computer-based approach to this repertoire. In this case, if we assume the (perhaps tasteless) chromatic alteration of all entries, we can view the point as a series of local intensifications of A major and minor. If we only grant the accidentals included by Elliott, the point starts off with little or no alteration, well within

51. See ‘Textual Commentary’ of BE 17, 152ff.
the natural minor mode. It then briefly intensifies A in bar 21 and quickly naturalises C and G to make way for the unaltered modality of the final point. Of course, there are any number of variant readings between these two interpretative poles, but untangling them is not the primary goal here.

3.2.5 Fifth point (bb. 23–end)

The final point is structurally interesting on two counts. First, the point opens with a four-note subject treated in *fuga* but with no overlapping between entries. The first five entries start on C, G, G, C and G respectively, a typical mapping-out of the tonal space that the point will occupy. However, bundled with the third and fifth entries is contrapuntal material that foreshadows the shape of the next entry (entry six) and every successive entry to the end of the fantasia. The original four note subject is extended by an extra descending step and the step from the third to the fourth is decorated consistently to the end. The second point of interest becomes apparent from this point on. From the sixth entry, subjects are introduced in pairs separated by the following number of minims: entries 6 and 7 separated by two minims, 8 and 9 separated by one, 10 and 11 separated by one and 12 and 13 separated by 3. Once again, the close of the point does not depend on *stretto*. Despite an initial reduction in entry separation between pairs, the point closes with the longest entry distance heard so far, making the upward scale in III bar 30 seem even longer than it would already (there being no such device so far) and reinforcing the full close on C.
3.3 Analysis of *Fantasia* a 4/g (BE 17/4)

3.3.1 First point (bb. 1–12)

The first point opens with the first subject on C in II with material in cantus firmus values in I which both appear verbatim two bars later in III and IV transposed down an octave, with a continuation line extending II to the half close in bar 7. The cantus firmus notes sit on G for two whole bars as the answer on G enters in I in bar 5. The ‘counter-subject’ cantus firmus material reappears in IV, this time embellished in bar 6 with the final skip contributing the bass part to the close. Two pairs of entries in stretto conclude the point, the first on G then D and the second, C then G. This point overlaps with next, there being no definitive close since bar 6 until bar 16.

The expected octave of the first answer is flexed to a descending perfect fifth, as a tonal answer, yet the subject continues as if it were a statement on C, the latter half grafted on to function as an ‘inner’ voice to the close. Other flexes important to the structure are the flex to a fourth of the descending skip in III bar 8 (avoiding a ninth with II owing to stretto) and a flex to a fourth to the same skip on the last entry on C in IV bar 10, which facilitates the overlap with tail of the subject in stretto (previously unheard in this combination).

3.3.2 Second point (bb. 12–16)

The subject of this tiny point is a four-note figure closely related in shape to the tail end of the subject of the first point. The last entry of this point in IV overlaps with the first entry of the third point in bar 16. Considering these four notes sequences as interlocks, after Grimshaw’s approach to Byrd’s early motets, is instructive. Grimshaw may have
classified this four note subject as a ‘peak-note cumulative 5th’ subject, since from base to apex, the subject delineates a fifth in its plain form. Part of the interlock approach is to determine a priori which interlocks at which transpositional levels produce viable counterpoint. Ignoring the almost trivial interlock at the distance of three minims (as seen between the first and second entries), and the interlock at the distance of two minims (not present in the point), Figure 5 shows the possible interlocks at the distance of one minim, with the upper voice leading, for the point.

There is in fact only one valid interlock for this subject in this arrangement—at the lower 5th. Yet in II bar 15 Byrd initiates an entry at the lower 7th, perhaps to complete the first-inversion D major sonority. The entry is both flexed and stretched. It must yield both to the proprieties of good counterpoint and the entry in III which gives the so-called ‘trivial’ interlock for the second time. Unsurprisingly, this entry has the highest average deformation metric for the point. Figure 6 shows the point in full.

3.3.3 Third point (bb. 16–27)

The subject of the third point is an ornamented descent through a perfect fifth. Plain-form candidates are entries are entries 1–4, 6, 7, 11 and 12. Most of the point (up to bar 24) consists of entries on either D or A. Transposition levels strongly imply cadence notes in sufficiently long entries and here the implication is underscored by Byrd’s persistent use of clausulae cantizans in the parts not presenting the entry. Here, entries on D imply a cadence on G and entries on A imply a cadence on D. The transposition level of cantus firmus note values regulates the transposition levels of the entries, and, where cantus firmus note values are not pervasive, their implied presence shapes the trajectory of continuation lines and shadowing parts. Figure 7 shows the point in full, with cut-out staves showing the material in cantus firmus values, the first and third
cut-outs from actual statements of the subject and cantus firmus material together, and the second, a conjectural transposition of the first, to match the transposition level of the second entry. The continuation line in I after the second entry takes the G and the A of the corresponding cantus firmus, while the shadow above the fourth entry in III is diverted from exact doubling at the third at the final note to supply the D of the corresponding cantus firmus. This point is also notable for the similarity of its entries (according to the deformation metric \( w = 6 \)). The point concludes with two entries in stretto which are plain-form candidates, both at the transposition level of D. This transposition level implies a cadence on G, and the point ends with the most assertive close in the fantasia up to this stage, the full close in bar 27.

3.3.4 Fourth point (bb. 27–34)

This brief point demonstrates the use of stretching to generate a rhythmically vital surface within the fuga framework. The result approximates homophony to some listeners, so much so that Neighbour incorrectly identified this section as ‘quasi-homophonic interlude.’\(^\text{52}\) Its rhythmic character is certainly remarkably different from that of the points which precede it, but its conception is essentially contrapuntal. The point is reproduced here in open score.

The material presented unequivocally for the first time in I is the subject matter for this point. All occurrences of this motivic shape are shown bracketed in Figure 8. The passage in III marked with an asterisk closely resembles an entry, but is in fact a composite ‘shadow.’ The first three notes shadow I at the third, as do the third, fourth and fifth shadow II. The line continues shadowing IV at the third above for the first three notes of the next bar. The subject does not appear in III until the second set of

\(^{52}\) Neighbour, 93ff.
entries in *stretto*.

The average deformation metric for each entry increases as this point progresses, the only exception being the restatement of the plain-form candidate (fourth entry). Furthermore, the successive deformation metric profile \((w = 5)\) is as follows: \(\langle 1, 0, 1, 1, 2, 5 \rangle\) demonstrating that the rate of deformation is also increasing as the point develops. That is, as the point progresses, each entry becomes less similar to the one immediately before it. Increased deformation rate comes with the decrease in distance between entries. The subject is increasingly deformed to produce viable counterpoint at the *stretto* distance of a minim.

3.3.5 Fifth point (bb. 34–46)

The longest point in this fantasia contains fourteen entries. Figure 9 summarises the location and starting note of each entry, as before. We are visually drawn to the entries connected by the fewest lines, that is, the fewest deformations. Of those, two are related by the interval of a perfect 5th. Once again, the rate of deformation increases as the point develops. The most deformed pairs of entries \((e_2 \text{ and } e_3, e_8 \text{ and } e_9, e_{12} \text{ and } e_{13})\) are separated by the shortest entry distance of the point of roughly a breve.

3.3.6 Sixth point (bb. 46–50)

This tiny point is rhythmically similar to the third point. The three-note subject (a descent through a minor third) is stretched (for example, bar 45) to conform to the controlling rhythmic figure of the bar. The final two entries (in II and III) in *stretto* imply a cadence on D, which is supplied with the raised third by the continuation line following the third last line (starting on E flat) neutering that line’s implication of a cadence on C (as in III bar 46, for example)
3.3.7  **Seventh point (bb. 50–end)**

Like the third, this point is notable for the similarity of its entries (according to the deformation metric \(w = 6\)). This is due to this point’s repetitious structure. In I and IV, bars 51–55 correspond to bars 56–60 almost precisely. The music of II and III is permuted for the first two bars of the repeated part, but the original part-order is restored in the second minim of bar 57. Both this point and the third point conclude with the only full closes of the fantasia. Reduced deformation activity and wholesale repetition provide stability and predictability both in these points which may, in other works, be shown to be strongly correlated to the presence of a full close. This hypothesis could be tested by applying the methods described above to the Byrd corpus, as it is encoded. A hierarchy of cadence formulas by strength (data either from contemporary theorists or from listener experiments) could be used to investigate if a relatively low amount of deformation correlates with the presence of relatively strong cadences; conversely, we can investigate if highly-deformed points correlates with the presence of relatively weak cadences. These are but some of many possibilities for analysis which are facilitated by the introduction of a subject deformation metric to the music of Byrd.
Conclusion

Having reviewed the state of the art of Byrd analysis, and with the benefit of seeing exactly how varied Byrd’s entries can become, it is clear that the case for the relevance of a strict idea of ‘imitation’ to Byrd’s points is weak. In fact, adopting a measurement of variation or ‘subject deformation’ that applies between any pair of entries categorically rejects the usefulness of a search for strict imitation. In so doing, we recognise that the task of the analyst is now to investigate the relationships where dissimilarity (referred to here as ‘deformation’) is the norm, rather than that of similarity as implied by the use of the term ‘imitation’. We have explained how a very simple subject deformation metric can be constructed, measuring flexing and stretching after Milsom’s recent definition of those operations in the *fuga* context. This metric as it is defined here is by no means the final stage in quantifying Byrd’s counterpoint. Further developments along a similar line could factor in any number of relevant musical considerations into the definition of the metric, with appropriate weightings. Questions of transposition level were only briefly touched upon, yet they could be included as important musical data when constructing an extension of the present metric. Metrical stresses are not considered in the present model, yet as the analyses above show, Byrd uses stretching to fascinating rhythmic effect. There is, thus, musical motivation for investigating how metrical relationships between parts (for example, syncopation) should be evaluated and included in a comprehensive quantitative model of Byrd’s counterpoint.
Quite apart from the methodological prospects, the model analyses have shed new light on items of Byrd’s consort repertoire which has not been systematically revisited in the English language since Neighbour’s monograph, which it is clear emanates from a different time indeed when considered alongside more recent writings on Byrd’s counterpoint. It appears that strong cadences are associated with reduced deformation activity (as in 4/g), and that flexing, once identified, can be shown to be coeval with accentuations of certain cadence notes (as in 3/C1) or with hints towards more distant tonal regions (as in the shades of A implied by subtle flexes in 3/C2). The data generated by the subject deformation metric can be used to enrich the often arid practice of entry diagrams with a new layer of information that invites analytical investigation, and which can be used to provide at-a-glance summaries of deformation activity. As the corpus of electronic transcription of works by Byrd grows, so grows the sample size for longitudinal studies of his music along investigative lines similar to those traced above. It is hoped in particular that the analyses herein have proven these lines of enquiry fruitful, and that further work may build and improve upon the simple ideas which have facilitated them, particularly those described in Chapter 2.

Early music analysis is still not an established practice. So too, perhaps, is this niche-within-a-niche of quantitative analysis of fuga. However, with the motivation that only a composer as prolific and compelling as Byrd provides, the most recent technology for parsing and analysing symbolic music transcriptions and the insights of a recent glut of studies into Byrd’s contrapuntal method, we might not have long to before this sub-field matures—and brings its community wisdom to the broader enterprise of early music analysis.
Appendix A

Glossary

*fuga subject*  The general rhythmic shape and melodic contour that predominates in a given point. Weakly analogous to the subject of a common practice fugue—but (importantly!) does not have to contain the exact same rhythms or intervals throughout a passage of *fuga*.

(subject) *entry*  A form of the subject which appears over the course of a passage of *fuga*. Such a passage will have several entries, rarely will they be all identical. Usually (but by no means necessarily) appears in a part after that part is resting.

*shadow*  Doubling of an entry at an imperfect consonance by another voice.

*continuation line*  Non-entry, non-shadow material *or* the purely contrapuntal material assigned to a fugal voice once it has presented the subject.

*flexing*  Modification of the melodic intervals of an entry, compared with any other.

*stretching*  Independent increase and reduction of one or many rhythmic note values of an entry, compared with any other.

*subject deformation*  Any operation that varies an entry, compared with any other, while retaining general rhythmic shape and melodic contour—its identity *qua*
subject. Flexing and stretching are both subject deformations.

**subject deformation metric** Here, a simple measurement between any two entries which enumerates the number of rhythmic and intervallic differences between them, with some exceptions (See 2.2.3 Limitations and adjustments).

**subject deformation metric window** A crucial parameter of the subject deformation metric which indicates how many notes (counting from the start) of each entry will be used in the calculation of the metric.

**plain-form candidate(s)** The entry or entries with the lowest total row- or column-wise sum of subject deformation metrics.

**successive deformation profile** A vector representing the subject deformation metric between successive entries in a point, starting with \( D(e_1, e_2), D(e_2, e_3), \ldots \).
Appendix B

Technical remarks

B.1 music21: A new and important development

Extracting statistical data from notation requires a tool that can interpret and process notational representations of the score. One such tool is a relatively recently developed Python module called music21. First released in 2010 and developed at MIT, music21 extends the Python programming language to provide programmatic representations of musical entities and functions which operate upon them.\textsuperscript{53} music21 is an extension of a programming language which has interpreters that are freely available for all platforms, including but not limited to UNIX, Linux, Microsoft Windows and OS X. This fact permits the use and integration of music21 into any Python application on any platform, including graphical user interfaces (GUIs). music21 is strongly recommended for facilitating the symbolic representation and manipulation of score entities and corpora.

B.2 Transcription methodology

The copy text used for the transcription process was Volume 17 of *The Byrd Edition, Consort Music*. This edition was edited by Kenneth Elliot and was published in 1976 by Stainer and Bell. The edition halves note values. Original note values have been restored in the transcription. Parts are labelled I, II, III etc. from the top voice down, as they appear in the print edition. Editorial accidentals have been included in the transcription. Variant readings annotated in the appendix to the edition have not been adopted in any case, for reasons of consistency. The scores were input by hand using Sibelius 6. Since music21 does not support the proprietary .sib file format, the score files were converted into MusicXML, for interchange purposes only. This can be achieved at no extra cost by first converting the .sib score to the MIDI format, which can is understood by Musescore (a free, open-source music typesetting suite) and saved as MusicXML. However, since the release of Sibelius 7, which natively supports conversion into MusicXML, this step is redundant if the latest version is available. The resultant MusicXML files were then loaded into memory on a case-by-case basis using the music21.converter.parse() function to be manipulated in Python. Source code relevant to the project is included below. It is intended to operate on any Python interpreter which supports music21 and has the latest version (and its dependencies) installed and correctly configured. At the time of writing, the latest version of music21 is v.1.4.0 and the latest version of Python which supports it is Python v.2.7. The platform used by the author was OS X Version v.10.8.2 (Darwin v.12.2.0), hence some very minor modifications to the code may be required for portability to other platforms.
B.3 Python/music21 implementation of subject deformation metric

This Python script takes a MusicXML and a window size as arguments and outputs a pretty-printed table of the subject deformation metrics for every entry input into the MusicXML file. The MusicXML file must have exactly one entry per stave, therefore, no empty staves. The starts of entries do not necessarily have to be vertically (rhythmically) aligned in the input score.

B.3.1 Program listing

```
''' Listing for def_metric.py '''
import math
import music21
import pprint
import collections
import texttable
import sys
PATH_TO_FILE = sys.argv[1]
WINDOW = int(sys.argv[2])
subject = music21.converter.parse(PATH_TO_FILE)
def getDataforPart(n):
    part = subject.parts[n]
    values = []
    pfs = part.flat.stripTies()
    for note in pfs.getElementsByClass('Note').notes:
        midinote = note.midi
        duration = note.duration.quarterLength
        values.append((midinote,duration))
    return values
def calculateDistanceScore(firstvalues, secondvalues, windowsize):
    score = 0
    for i in range(0, windowsize):
        try:
            firstinterval = firstvalues[i+1][0] - firstvalues[i][0]
            secondinterval = secondvalues[i+1][0] - secondvalues[i][0]
        except:
            return 0.1
        diatonicflex = ((firstinterval == 1 and secondinterval == 2) or
                        (firstinterval == 2 and secondinterval == 1))
        if firstinterval != secondinterval and not diatonicflex:
            score += (firstinterval + secondinterval) / 2
    return score
```
score +=1
if firstvalues[i][1] != secondvalues[i][1] and i != 0:
    score += 1
return score

numberofparts = len(subject.parts)
valueslist = []

for i in range(0, numberofparts):
    valueslist.append(getDataforPart(i))

tab = texttable.Texttable(max_width=0)
tab.header(["\*" ] + [i for i in range(0, numberofparts)]+["T"]) 
for i in range(0, numberofparts):
    row = [i]
    running = 0
    for j in range(0, numberofparts):
        score = calculateDistanceScore(valueslist[i], valueslist[j], WINDOW)
        row.append(score)
        running += math.floor(score)
    row.append(running)
    tab.add_row(row)

print "Deformation chart for %s" % PATH_TO_FILE
print "Window size: %d" % WINDOW
print tab.draw()}
### B.3.2 Usage and example output

**Usage example**

```bash
gnome ~% python def_metric.py ./Scores/Entries/1-fa3-i/XML/1-fa3-i-3.xml 9
```

Deformation chart for ./Scores/Entries/1-fa3-i/XML/1-fa3-i-3.xml

Window size: 9

```
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| * | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | T |
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| 0 | 0 | 0 | 0 | 0 | 2 | 0 | 3 | 1 | 0 | 3 | 3 | 6 | 0 | 0 | 18 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 1 | 0 | 0 | 0 | 0 | 2 | 0 | 3 | 1 | 0 | 3 | 3 | 6 | 0 | 0 | 18 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 2 | 0 | 0 | 0 | 0 | 2 | 0 | 3 | 1 | 0 | 3 | 3 | 6 | 0 | 0 | 18 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 3 | 0 | 0 | 0 | 0 | 2 | 0 | 3 | 1 | 0 | 3 | 3 | 6 | 0 | 0 | 18 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 4 | 2 | 2 | 2 | 2 | 0 | 2 | 1 | 3 | 2 | 5 | 5 | 8 | 2 | 2 | 38 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 5 | 0 | 0 | 0 | 0 | 2 | 0 | 3 | 1 | 0 | 3 | 3 | 6 | 0 | 0 | 18 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 6 | 3 | 3 | 3 | 3 | 1 | 3 | 0 | 4 | 3 | 6 | 6 | 8 | 3 | 3 | 49 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 7 | 1 | 1 | 1 | 1 | 3 | 1 | 4 | 0 | 1 | 3 | 3 | 6 | 1 | 1 | 27 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 8 | 0 | 0 | 0 | 0 | 2 | 0 | 3 | 1 | 0 | 3 | 3 | 6 | 0 | 0 | 18 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 9 | 3 | 3 | 3 | 3 | 5 | 3 | 6 | 3 | 3 | 0 | 0 | 7 | 3 | 3 | 45 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 10 | 3 | 3 | 3 | 3 | 5 | 3 | 6 | 3 | 3 | 0 | 0 | 7 | 3 | 3 | 45 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 11 | 6 | 6 | 6 | 6 | 8 | 6 | 8 | 6 | 6 | 7 | 7 | 0 | 6 | 6 | 84 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 12 | 0 | 0 | 0 | 0 | 2 | 0 | 3 | 1 | 0 | 3 | 3 | 6 | 0 | 0 | 18 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
| 13 | 0 | 0 | 0 | 0 | 2 | 0 | 3 | 1 | 0 | 3 | 3 | 6 | 0 | 0 | 18 |
+----+---+---+---+---+---+---+---+---+---+---+----+----+----+----+----+
```

gnome ~%
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The consort fantasias of William Byrd: The application of a new quantitative technique to describe *fuga* subject deformation

FIGURES AND TABLES

Eamonn Bell
Trinity College, Dublin

April 2013
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